2008 High School Mathematics Core Comprehensive Materials Review & Recommendations Report

Initial Recommendations

Updated January 15, 2009

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Revision History

Date	Version Notes	Updated By
1/6/09	Preliminary Draft completed. All results subject to change and verification.	Porsche Everson
1/15/09	Initial Recommendations Draft completed. Incorporated changes based upon feedback from Math Panel. Added section on initial recommendations.	Porsche Everson

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1 Executive Summary

1.1 Introduction/Purpose

The purpose of this document is to describe the process and outcomes from the 2008 Mathematics Core/Comprehensive Instructional Material Review for high school. The report contains information about the entire process, as well as statistical results from the review.

It is important to note that successful mathematics programs may exist with virtually all of the reviewed curricula. While instructional materials matter, other factors contribute to the success of students in Washington state learning mathematics. Those factors include quality of instruction, parent involvement, available supports and myriad other aspects.

While the recommended curricula will ultimately receive the bulk of attention within this report, it also provides other key results as well. These results include:

- **Information on all curricula materials reviewed:** Districts who currently use instructional material *not* in the top three recommendations will find this report valuable. It contains detailed, specific information on how all programs reviewed meet the newly revised 2008 Washington State High School Mathematics Standards. Instructors, coaches, curriculum specialists and administrators can easily see how their materials line up against the standards, course by course, and identify areas where supplementation may be needed. *No one set of instructional materials matches the new standards completely; each one will need some augmentation, even those that are recommended.*

- **Support to districts in evaluating instructional materials:** Finally, local districts can use the rich set of information contained within to evaluate a wide variety of textbooks based upon factors they deem important, to help them make decisions in future regarding mathematics textbook adoptions.

Some words of caution are necessary. Reviews of instructional materials represent a point in time, in a continuously evolving process. New versions will rapidly supplant those reviewed herein.

In general, there are multiple versions of instructional materials in use by districts across the state. This review process examined only one version of each program; typically the most recently copyrighted version. Readers should be aware that older versions of the programs would likely have different results. Many districts are using older versions of these programs.

The existing programs were evaluated against newly revised standards. No publisher has had the chance to update their material to produce a new version since the high school standards were released in July 2008. This review simply provides a baseline comparison, from which publishers can adapt their material to be more closely aligned with the revised Washington standards.

Finally, it should be noted that there are two sets of standards for high school math. The first tracks the traditional Algebra 1, Geometry, and Algebra 2 series. The second is a re-ordered set of the same standards for Integrated Math 1, 2 and 3. Integrated Math is a more recent development in mathematics education, and does not share the same approach to ordering the standards by course level. Thus, while the more mature Algebra series publishers align to the course-by-course standards, Integrated Math products align to the entire series of standards and there is variability among the publishers as to when the standards are met in the series. One of the instructional materials review outcomes was to identify where the standards were typically met in the submitted products.

1.2 Scope and Background

As per 2007 and 2008 Legislation, OSPI is required to recommend no more than three basic mathematics curricula at the elementary, middle and high school grade spans to the State Board of Education (SBE) within six months of the adoption of the revised standards for their "review and comment". [1] The high school standards were adopted on July 30, 2008. In undertaking the process for making the recommendations, OSPI elected to conduct an instructional materials review that evaluated published core/comprehensive high school mathematics instructional materials using the 2008 Revised Washington State Mathematics Standards and other factors. The resultant data was used to inform the selection process for the recommendations.

Once OSPI makes the initial recommendations to the SBE, the SBE has two months to provide official comments and recommendations. The superintendent of public instruction shall make any changes based upon SBE's comments, and adopt the recommended curricula.

In addition, 2008 Second Substitute House Bill (2SHB) 2598 indicates that appropriate diagnostic and supplemental materials "shall be identified as necessary to support each curricula." OSPI is engaging in a Mathematics Supplemental Materials Review to meet this objective for grades K-12. The results from the K-12 Supplemental Review will be released in a separate report. To address providing support for the selection of mathematics diagnostic materials, OSPI has developed a Diagnostic Assessment Guide that was made available to school districts in late fall 2008 and provides information on diagnostic assessment materials available in mathematics, reading, writing, and science. This work began in 2007 in response to 2007 Senate Bill 6023.

1.3 Contributing Stakeholders

Many individuals and groups contributed to the development of the instructional materials review process, instrument design, materials review, data analysis and development of the report.

[1] See 2008 Second Substitute House Bill (2SHB) 2598.

- Instructional Materials Review (IMR) Advisory Group – A group of 22 curriculum specialists, mathematics educators, mathematicians, math coaches, educational service district math coordinators, and district administrators from all over the state who have experience in curriculum reviews.
- State Board of Education Math Panel – Educators, mathematicians, parents, university faculty, advocacy group and business representatives who were actively involved in providing input on the revised mathematics standards and have key knowledge on effective, research-based mathematics instruction.
- Materials Reviewers – 28 individuals from around the state representing a diverse coalition of professionals and lay people, including math educators, math coaches, curriculum specialists, parents, business people, advocacy groups, district administrators and mathematicians.
- OSPI Staff – educational leaders, mathematics specialists, and support staff.
- National Experts and External Leaders – Individuals who shared their background and experience with state-level materials review and adoption processes. It is important to note that these individuals contributed information about their state-level materials review and adoption processes. Some but not all of their ideas were incorporated into the Washington process. Inclusion of their names does not imply that they endorsed the results contained within this report.
  - Charlene Tate-Nicols (Connecticut)
  - Jonathan Weins, Drew Hinds (Oregon)
  - James Milgram (California)
  - Jane Cooney (Indiana)
  - Charlotte Hughes (North Carolina)
  - George Bright (Washington)
  - Jim King (Washington)

1.4 Process Overview

The 2008 Core/Comprehensive Mathematics Instructional Materials Review involved very high stakes outcomes, particularly the selection of no more than three basic curricula recommendations in the elementary, middle and high school grade spans (K-5, 6-8, and 9-11). Thus, the project processes and controls were designed to be rigorous, transparent, inclusive and reliable. Hundreds of professionals contributed to the success of the project during its multiple phases.

Phase	Process Steps
Design Review Instrument and Process	• Sought input from multiple stakeholder groups, including IMR Advisory Group and SBE Math Panel - Iterative development process with two full cycles of feedback • Research-based foundational resource materials included 2008 Washington Revised Math Standards, National Mathematics Advisory Panel Foundations for Success (NMAP), and National Council of Teachers of Mathematics (NCTM) Curriculum Focal Points • Used process feedback from other states which have

Phase	Process Steps

	successfully completed curriculum reviews to design instrument and review process.
	• Outcomes included:
	  o Two review instruments (Content/Standards Alignment and Other Factors)
	  o Proposed threshold process for deriving final recommendations
	  o Proposed weighting for instrument scales

Solicit Publisher Involvement	• All publishers invited to submit materials • Publisher's conference held to address questions and clarify submission process • Question and Answer document disseminated widely and updated throughout period prior to the review • Publishers provided alignment worksheet to show where their materials aligned to revised state standards • Publishers submitted multiple sets of materials for review week
Select IMR Review Committee	• Application materials widely distributed statewide to school districts and education stakeholder groups, including math educators, curriculum specialists, advocacy groups • Objective review and scoring of each application by two independent reviewers using a common review instrument • Selections based upon score and having sufficient variation in expertise among reviewers (educators, mathematicians, community representatives, curriculum specialists, administrators, parents, etc.)
Review Instructional Materials	• Rigorous process for controlling inventory, during publisher check-in, reviewer check-in/out, and publisher check-out • Reviewers received full-day training in high school standards • Trained reviewers in how to use the scoring instruments • Performed real-time data entry • Performed variance checks and corrective training to reduce variance and increase inter-rater reliability • Independent reviews of materials • Five or more reads on all of the material • Random assignment of materials to reviewers • Twice-daily progress monitoring • Process improvement checks daily
Analyze Data	• Exploratory data analysis by two independent statisticians • Quality control checks comparing random 10% of score sheets to electronic data to ensure accuracy of data entry

Phase	Process Steps
	and extract processes • Rigorous design of statistical tests, validated by expert statistician • Presentation of results in easy to read tabular and graphical format
Present Preliminary Results	• Followed legislatively mandated protocol and timeline • Presented preliminary results to State Board of Education Math Panel • Sought advice from SBE Math Panel on the analysis, recommendations and process • Presented preliminary results to legislators, districts, publishers, review participants, and public
Select Recommendations	• Sought advice from the State Board of Education • Used process and resultant data to inform the recommendation selections
Provide Support to Districts	• Communicated with districts about what information they need, and included that information in the preliminary report • Provided key information on how all mathematics curricula reviewed aligns to 2008 revised Mathematics Standards • Will provide information about supplemental programs (in a separate report) designed to augment reviewed curricula to better meet Washington standards.

1.5 Findings

1.5.1 Data

The following tables show the overall ranking for all core comprehensive programs submitted for review. The scaled category score is the rating value expressed as a proportion of all possible points in the category. The scale value is calculated by averaging the raw scores in a category, then dividing by the maximum scale value to obtain a scaled average. Each category was assigned a weight, as described elsewhere in this report. The weights were used to derive a final composite score.

The final composite score is calculated using the formula:

$$\sum (Average\ Scale\ Score)(Scale\ Weight)$$

Table 1. Ranked list of all core/comprehensive Algebra 1 and 2 series reviewed, ordered by final composite score.

Overall Ranking for All Algebra 1 and 2 Series							
Program	**Content/ Standards Alignment**	**Program Organization and Design**	**Student Learning**	**Assessment**	**Instructional Planning and Professional Support**	**Equity and Access**	**Final Score**
Weights	**70%**	**9%**	**7.5%**	**5%**	**4.5%**	**4%**	
Discovering – Algebra	0.863	0.897	0.870	0.822	0.837	0.758	0.859
Holt Algebra	0.841	0.821	0.800	0.795	0.777	0.864	0.832
Glencoe McGraw-Hill Algebra	0.823	0.827	0.836	0.807	0.826	0.742	0.821
PH Math Algebra	0.833	0.770	0.776	0.750	0.754	0.783	0.814
CPM Algebra	0.751	0.836	0.867	0.845	0.803	0.601	0.768
McDougal Littell Algebra	0.786	0.661	0.658	0.716	0.595	0.763	0.752
CME Algebra	0.739	0.773	0.755	0.670	0.716	0.545	0.731
Cognitive Tutor Algebra	0.735	0.709	0.703	0.697	0.640	0.485	0.714
CORD Algebra	0.705	0.757	0.733	0.575	0.742	0.511	0.699
PH Classics (Foerster) Algebra	0.709	0.653	0.714	0.531	0.573	0.287	0.672
PH Classics (Smith) Algebra	0.692	0.571	0.612	0.607	0.521	0.575	0.658
MathConnections Algebra	0.528	0.644	0.654	0.279	0.670	0.295	0.532
Average	0.746	0.737	0.744	0.667	0.699	0.594	0.733

Table 2. Ranked list of all geometry programs reviewed, ordered by final composite score.

Overall Ranking for All Geometry Programs							
Program	Content/ Standards Alignment	Program Organization and Design	Student Learning	Assessment	Instructional Planning and Professional Support	Equity and Access	Final Score
Weights	**70%**	**9%**	**7.5%**	**5%**	**4.5%**	**4%**	
Holt Geometry	0.860	0.828	0.794	0.778	0.861	0.824	0.847
McDougal Littell Geometry	0.850	0.820	0.813	0.875	0.808	0.833	0.843
Glencoe McGraw-Hill Geometry	0.847	0.800	0.800	0.851	0.786	0.722	0.832
PH Math Geometry	0.854	0.800	0.747	0.717	0.767	0.767	0.827
CORD Geometry	0.810	0.872	0.822	0.590	0.819	0.546	0.795
Discovering – Geometry	0.783	0.793	0.787	0.708	0.767	0.700	0.776
Cognitive Tutor Geometry	0.699	0.833	0.817	0.826	0.854	0.630	0.730
CPM Geometry	0.744	0.757	0.776	0.637	0.679	0.492	0.729
CME Geometry	0.625	0.617	0.639	0.625	0.583	0.370	0.613
MathConnections Geometry	0.512	0.633	0.644	0.410	0.688	0.324	0.528
Average	0.756	0.774	0.764	0.700	0.759	0.613	0.750

Table 3. Ranked list of all integrated math programs reviewed, ordered by final composite score when treated as individual courses

Overall Ranking for All Comprehensive Integrated Math Programs when Treated as Individual Courses							
Program	Content/ Standards Alignment	Program Organization and Design	Student Learning	Assessment	Instructional Planning and Professional Support	Equity and Access	Final Score
Weights	70%	9%	7.5%	5%	4.5%	4%	
Core Plus Math	0.671	0.771	0.760	0.701	0.799	0.535	0.688
SIMMS Math	0.656	0.763	0.683	0.589	0.672	0.476	0.658
Interactive Math Program	0.490	0.758	0.725	0.406	0.724	0.493	0.538
Average	0.606	0.764	0.723	0.565	0.732	0.501	0.628

Table 4. Ranked list of all integrated math programs reviewed, ordered by final composite score when treated as a series as a whole.

Overall Ranking for All Comprehensive Integrated Math Programs when Treated as a Series							
Program	Content/ Standards Alignment	Program Organization and Design	Student Learning	Assessment	Instructional Planning and Professional Support	Equity and Access	Final Score
Weights	70%	9%	7.5%	5%	4.5%	4%	
Core Plus Math	0.802	0.771	0.760	0.701	0.799	0.535	0.780
SIMMS Math	0.710	0.763	0.683	0.589	0.672	0.476	0.696
Interactive Math Program	0.609	0.758	0.725	0.406	0.724	0.493	0.621
Average	0.707	0.764	0.723	0.565	0.732	0.501	0.699

Table 5 shows the 95% confidence intervals for all comprehensive Algebra 1 and 2 series. The composite score represents the sum of the weighted scaled averages for each scale. See *5.9 Standard Error Calculations* for additional detail.

The charts in this section display the program final composite scores and their confidence intervals. Programs with overlapping confidence intervals should be considered as not being significantly different. Programs with non-overlapping confidence intervals can generally be considered to be statistically different in their ratings. However, when multiple tests are performed and we adjust for multiple comparisons, some non-overlapping intervals may be found to be not statistically different. Thus, the visual chart provides a quick check, but readers should rely on the specific test outcomes to determine statistical significance.

Table 5. Confidence interval values for all Algebra 1 and 2 series reviewed.

Program	Composite Score	Std. err.	95% CI Lower	95% CI Upper
Discovering - Algebra	0.859	0.009	0.842	0.876
Holt Algebra	0.832	0.009	0.815	0.849
Glencoe McGraw-Hill Algebra	0.821	0.008	0.804	0.837
PH Math Algebra	0.814	0.009	0.796	0.831
CPM Algebra	0.768	0.012	0.745	0.791
McDougal Littell Algebra	0.752	0.010	0.732	0.771
CME Algebra	0.731	0.011	0.710	0.753
Cognitive Tutor Algebra	0.714	0.009	0.696	0.733
CORD Algebra	0.699	0.011	0.677	0.721
PH Classics (Foerster) Algebra	0.672	0.011	0.650	0.695
PH Classics (Smith) Algebra	0.658	0.010	0.638	0.679
MathConnections Algebra	0.532	0.011	0.511	0.553

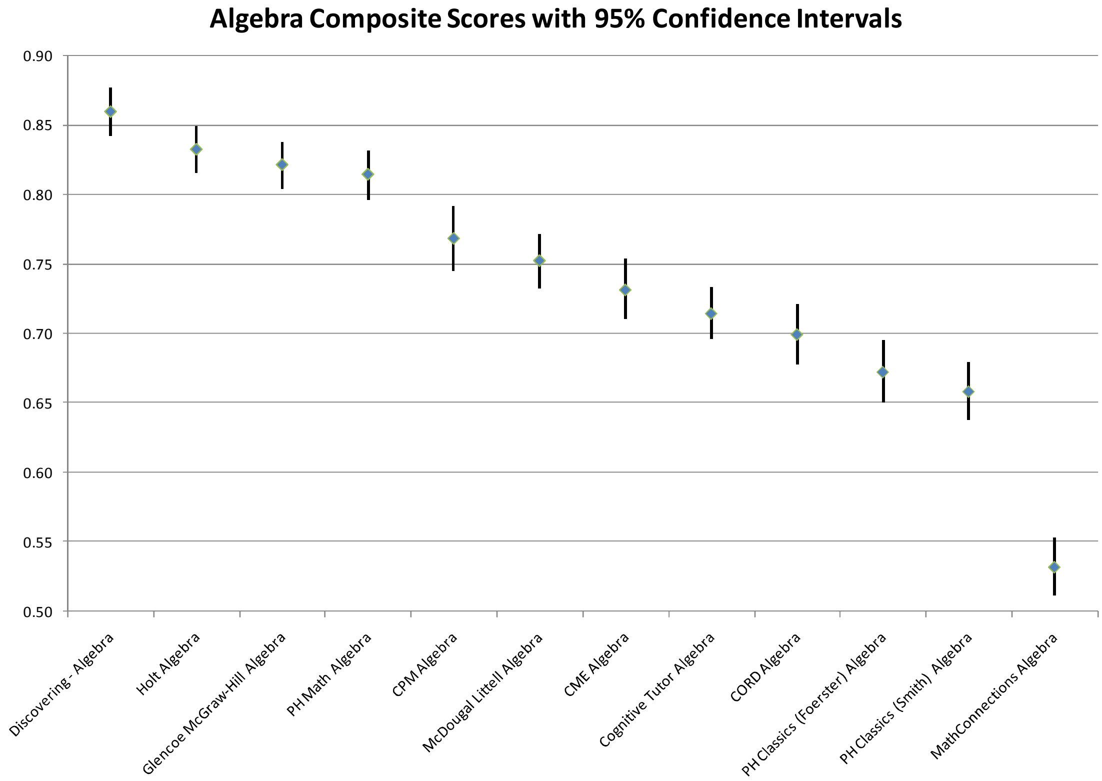

Figure 1. 95% confidence intervals for core/comprehensive Algebra 1 and 2 series.

The geometry results are presented below.

Table 6. Confidence interval values for all geometry programs reviewed.

Program	Composite Score	Std. err.	95% CI Lower	95% CI Upper
Holt Geometry	0.847	0.010	0.828	0.866
McDougal Littell Geometry	0.843	0.013	0.818	0.868
Glencoe McGraw-Hill Geometry	0.832	0.009	0.813	0.850
PH Math Geometry	0.827	0.012	0.803	0.851
CORD Geometry	0.795	0.014	0.769	0.822
Discovering - Geometry	0.776	0.014	0.748	0.804
Cognitive Tutor Geometry	0.730	0.015	0.700	0.761
CPM Geometry	0.729	0.013	0.704	0.755
CME Geometry	0.613	0.014	0.586	0.641
MathConnections Geometry	0.528	0.015	0.499	0.557

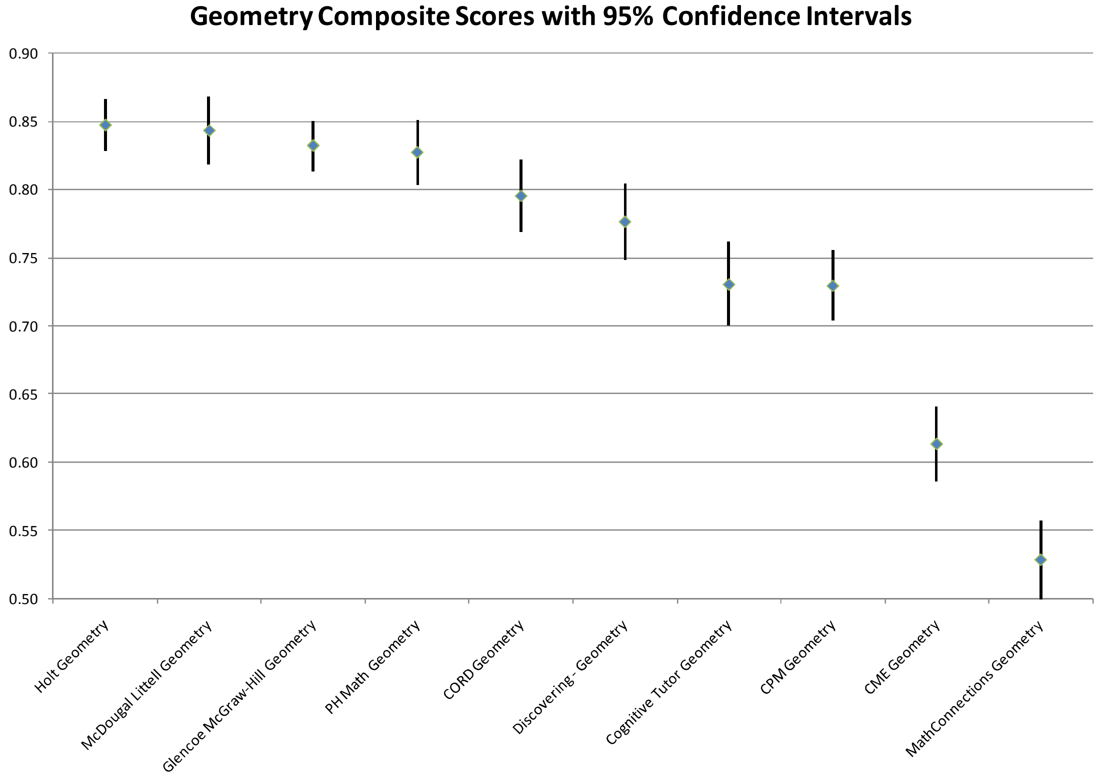

Figure 2. 95% confidence intervals for core/comprehensive geometry programs.

The following tables and graphs show the results for the Integrated Mathematics curricula.

Table 7. Confidence interval values for all integrated mathematics programs reviewed when treated as individual courses.

Program	Composite Score	Std. err.	95% CI Lower	95% CI Upper
Core Plus Math	0.688	0.009	0.670	0.706
SIMMS Math	0.658	0.009	0.639	0.676
Interactive Math Program	0.538	0.010	0.518	0.558

Table 8. Confidence interval values for all integrated mathematics programs reviewed when treated as an entire series.

Program	Composite Score	Std. err.	95% CI Lower	95% CI Upper
Core Plus Math	0.780	0.008	0.764	0.796
SIMMS Math	0.696	0.009	0.678	0.714
Interactive Math Program	0.621	0.010	0.601	0.642

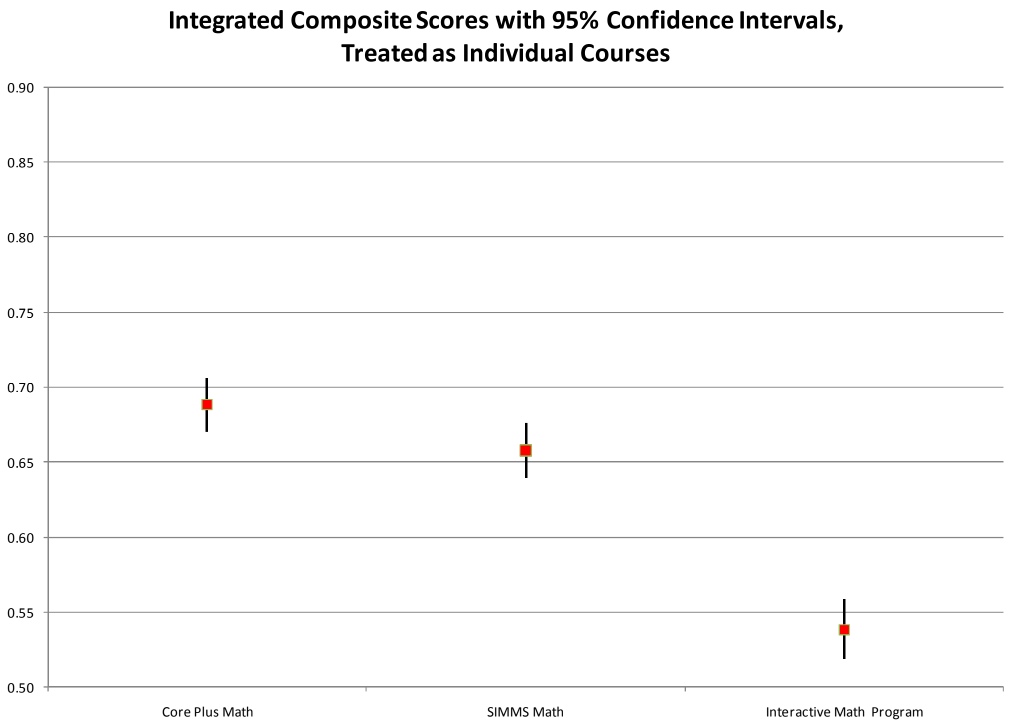

Figure 3. 95% confidence intervals for core/comprehensive integrated math programs when treated as individual courses. (Score reductions were applied when standards were found in alternate courses.)

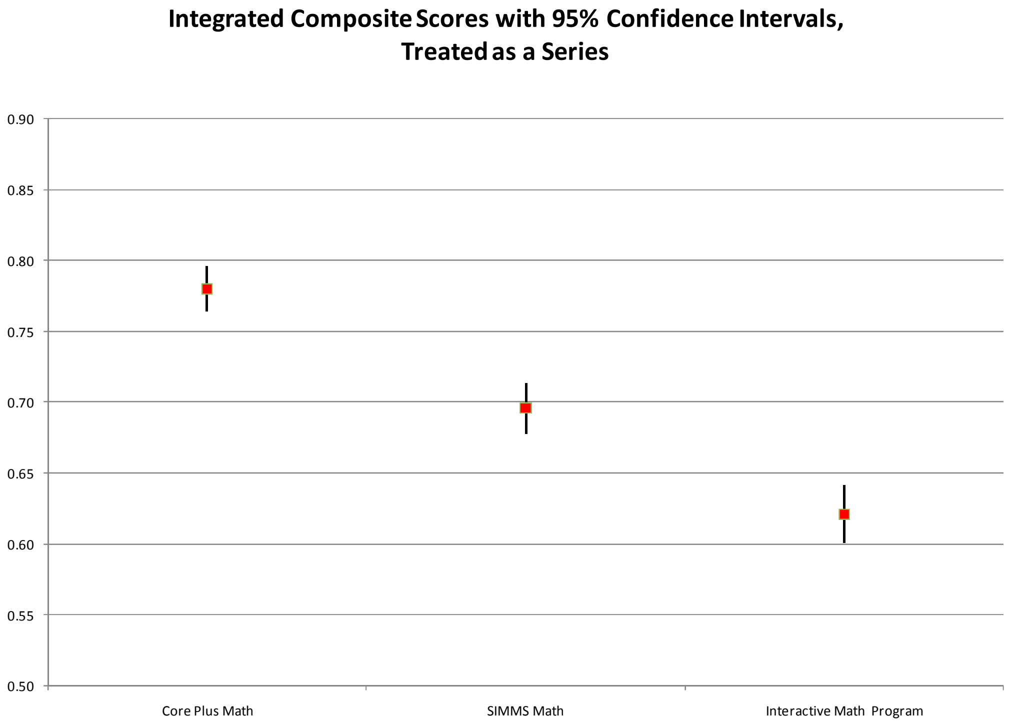

Figure 4. 95% confidence intervals for core/comprehensive integrated math programs when treated as a series. (No reductions for standards found in alternate course levels.)

1.5.2 Publisher Bundle Comparison

One interesting comparison to make is how do traditional and integrated series match up to each other when compared as a three-year series? The following chart and graph show the results. Note that both the Traditional and Integrated products are measured as a series, not as individual courses in this comparison. Thus, there is no reduction in the content score for standards found outside the expected course level.

Table 9. Traditional and Integrated three-year publisher bundles, in rank order, treated as a series (without a reduction in score for standards that are met at alternate course levels).

				95% CI	
Program	**Composite Score**	**Std. err.**	**Type**	**Lower**	**Upper**
Holt	0.838	0.007	Traditional	0.825	0.851
Discovering	0.835	0.007	Traditional	0.820	0.849
Glencoe McGraw-Hill	0.826	0.006	Traditional	0.814	0.838
PH Math	0.820	0.007	Traditional	0.806	0.834
McDougal Littell	0.783	0.008	Traditional	0.767	0.799
Core Plus Math	0.780	0.008	Integrated	0.764	0.796
CPM	0.755	0.009	Traditional	0.738	0.772
CORD	0.739	0.009	Traditional	0.722	0.756
Cognitive Tutor	0.723	0.008	Traditional	0.706	0.739
SIMMS Math	0.696	0.009	Integrated	0.678	0.714
CME	0.692	0.009	Traditional	0.674	0.709
Interactive Math Program	0.621	0.010	Integrated	0.601	0.642
MathConnections	0.562	0.009	Traditional	0.545	0.579

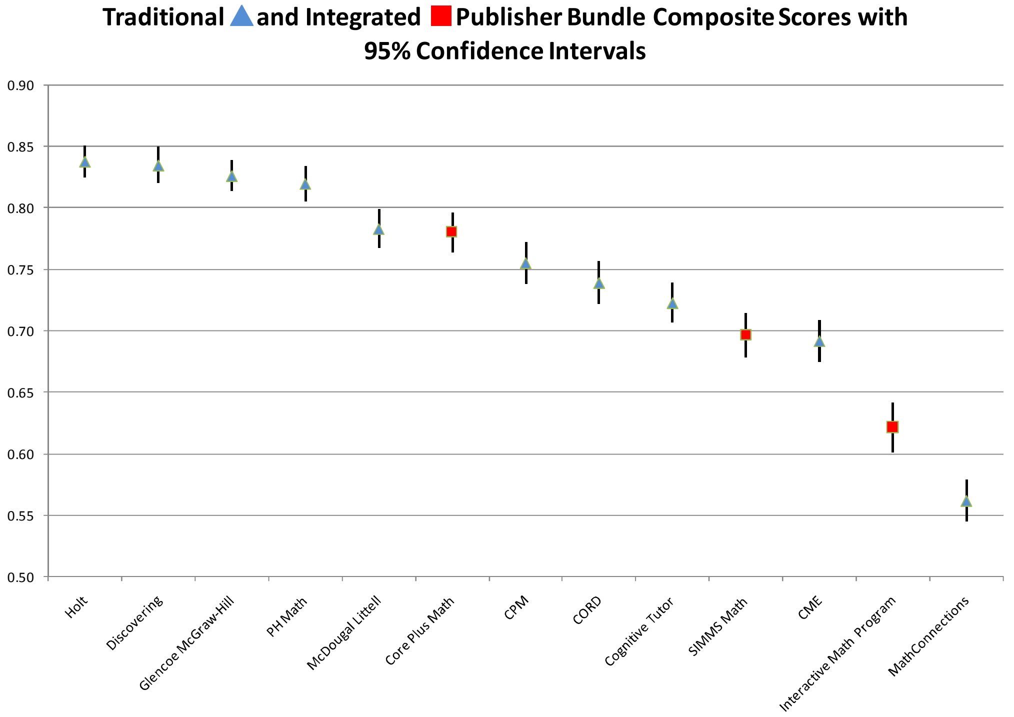

Figure 5. Comparison of both Traditional and Integrated three-course series, treated as a whole series, not individual courses. A traditional bundle is Algebra 1, Geometry and Algebra 2. An Integrated bundle is Math 1, 2 and 3.

1.5.3 Course/Standards Placement

The purpose of this section is to describe how well existing courses match up with the new 2008 Washington Mathematics standards.

Almost 4% of the time, Algebra 1 standards were found in the Algebra 2 text, or vice versa. This was mostly in quadratic and exponential functions.

In Integrated Math, almost 30% of the standards were met in a course above or below the level for the specific performance expectation. The concentrated areas for Integrated Math were quadratic functions, conjectures and proofs, volume and surface area.

Algebra 1 and 2 are well established courses, which haven't changed much in recent years. There is a high degree of agreement among publishers, mathematicians, and educators about what constitutes an Algebra 1 course versus an Algebra 2 course. In contrast, Integrated Math is newer, and there is less agreement about what constitutes a Math 1 course, versus a Math 2 or 3 course. There is more variability among publishers in terms of content placement. Further, there is no national agreement on the placement of standards within Integrated Math. Finally, balancing the standards among the three integrated courses was a key design element for the recent standards revision project.

In the initial data analysis approach for this project, we allocated ½ of the raw score for a standard if it was met in an alternate course level. Thus, if a publisher's program fully met an Algebra 1 standard in the Algebra 2 text, it received ½ of the raw score of 3, or 1.5. This focuses the data on individual courses, and how well each specific course aligns to its respective performance expectations.

Because of the large number of standards found in alternate course levels within the Integrated Math series, we elected to present results both treated as individual courses, and as an entire series (without a reduction in score if the standard was found in an alternate course level). This will allow readers to see results both ways. Unless otherwise noted, the data shown in tables and charts is measuring results for individual courses, meaning that ½ of the raw score was allocated if the content was found in an alternate course level.

The results for the Algebra 1 and 2 series are unchanged, regardless of whether the grade dip adjustments are applied or not. However, there is significant difference within the Integrated Math programs, both in terms of which programs exceed the minimum content threshold, and the overall content and composite scores for all integrated programs.

More work remains to analyze which standards were most commonly met in course levels above or below the expected course.

1.5.4 Online Availability

One of the further requirements of HB 2598 was for at least one of the recommended curricula at each level to be available online. As part of finalizing this report and determining the recommendations, we checked the online availability of the top-ranked programs. We specifically inquired as to the online availability of the materials that were reviewed in hard copy for their alignment as part of our review. The online availability of instructional materials typically takes the form of access by teachers, students, and parents to a PDF version of the applicable materials.

Districts typically negotiate costs of licenses to access the online materials during the purchasing process. Most of the licenses were for a renewable six year period, and offered seats based upon the number of student textbooks purchased. Once purchased, most products had significant flexibility in assigning access rights to the online material.

The table below denotes detail about the materials available online for each of the top ranked programs. Please note that while supplemental materials may be available, OSPI may not have reviewed them for their alignment with the revised standards.

Table 10. Online availability for the top-ranked programs.

Program Name	Core Materials Available Online	Supplemental Materials Available Online	Teacher Materials Available Online	URL
Holt Mathematics	Yes	Yes	Unknown	
Discovering	Yes	Yes	Yes	www.keypress.com

Program Name	Core Materials Available Online	Supplemental Materials Available Online	Teacher Materials Available Online	URL
Core Plus Math	Yes	Yes	No	www.mcgraw-hill.com[2]

1.5.5 Comments

Reviewers had the opportunity to provide optional comments on each of the programs they reviewed. Their comments are included in a separate companion document, available on the OSPI web site.

Many individuals commented on the K-8 report. Because the process and methodology are so similar for the high school report, a summary of the most common comments and responses are presented below.

Comment	Response
Will districts be required to adopt these materials?	No. These are recommendations only. Districts are free to select any program they feel best meets the needs of their students. Districts may find this report particularly helpful, along with the accompanying data set as they make their curriculum decisions. The State Board of Education is considering a proposal that would mandate use of one of the recommended programs if the district is consistently failing to meet expectations.
There are other ways to analyze the data. Why didn't you use method _______?	We agree that there are many methods that could have been used to analyze the data. Prior to collecting data, during the design of the process, we considered several possibilities and selected t-tests with multiple comparisons for our primary test statistic. Post hoc changes in methodology are risky; and lead to concerns that the analyst is seeking specific results. Thus, we continued to present results with our planned analysis approach.
What happens if some programs are tied with the top three?	The legislation mandates that OSPI select no more than three programs at each level. Thus, if there are ties, OSPI must still select no more than three. We will note in the report where ties exist.
My district is using program _______, which is not in the top three. What will OSPI do to help us out?	OSPI will be providing a report on available supplemental material and how well the materials align to state standards. In addition there are several tables and charts that show how each program performs, for specific Performance Expectations and mathematics Core Content within the

[2] See http://www.glencoe.com/sites/washington/teacher/mathematics/index.html for specific references to the online version of Core Plus Math.

Comment	Response
	standards. This information will help districts identify areas where supplementation is needed in existing programs.
Will the state be funding textbook purchases, based on these results?	At this point, there is no funding identified for textbook purchases based on these results.
I believe some standards are more important than others, why are they all weighted the same?	Most individuals feel that some standards are more important than others. However, there is no agreement among stakeholder groups about which are the most important. OSPI elected to take a neutral stance, and weigh all the standards the same for the purposes of collecting and analyzing the data.
There is some concern about program placement in the rank order, where individuals thought a program should have appeared higher or lower than it did.	It should be noted that the vast majority of the reviewed programs had a very reasonable correlation to the newly revised state standards and the other factors measured. Each program-course had four independent reads. Overall, the scores are good, and just because a program falls in the middle of the pack doesn't mean it isn't a viable choice, depending upon the district's needs. Most states have a textbook evaluation process that sets a basic threshold and all programs that meet or exceed that basic threshold can be considered for purchase. Washington state is unique in providing no more than three recommendations. If this review had been conducted in a more traditional manner, almost every single program would likely be in the pool of approved materials.

1.6 Recommendations

OSPI reviewed the results from the instructional materials review and comments from the Math Panel, and makes the following initial recommendations.

Table 11. Initial recommendations for publisher bundles in traditional and integrated series.

Publisher Bundle	Type of Program	Final Composite Score[3]	Overall Rank
Holt Mathematics	Traditional (A/G/A)	0.838	1st
Discovering	Traditional (A/G/A)	0.835	2nd
Core Plus Math	Integrated	0.780	Tied for 5th/6th

The following observations are worth noting when considering the recommendations:

- There is a strong depth of field in the traditional Algebra 1, Geometry and Algebra 2 series. Most products have high alignment to the 2008 Washington math standards, exceed the content/standards threshold established in this process, and have high scores on other scale factors. Programs such as Glencoe McGraw-

[3] Composite score is calculated for the series as a whole, and does not take into account reductions in scores for standards met above or below the expected course level.

Hill, Prentice Hall Math, and McDougal Little Math are viable products, even if they did not receive a recommendation in this process.

- About forty percent of the districts in Washington use an integrated series for high school math, either alone or in combination with a traditional series. Because of the broad usage of integrated programs across the state, OSPI is including a recommendation for the top ranked integrated program, bypassing some traditional programs that have higher composite scores. However, OSPI has great concern over the high number of instances in the integrated curricula where standards were met in course levels either above or below the level specified in the standards, and urges districts to carefully consider the impact of the new end of course assessments for both traditional and integrated courses in their curriculum adoption decisions.
- Within the traditional series Holt and Discovering, there are two different pedagogical approaches. Holt is stronger in development of algorithms and standard methods. The Discovering series emphasizes real-world problems, use of technology, and interactive learning.

1.6.1 Conclusion

The legislature directed OSPI to recommend no more than three programs at each level, elementary, middle and high school. The recommended programs at the high school level are closely aligned with the 2008 Washington Mathematics Standards, are mathematically sound and collectively provide a variety of instructional approaches. However, no program aligned completely to the new standards, and all will need some degree of supplementation. OSPI is engaging in a supplemental review and will provide an ancillary report that highlights supplemental products that provide a good fit for these recommended programs and others in common use around the state.

2 Project Process

2.1 Review Instrument Development

This section describes the process by which the review instrument and weights were developed. It also includes the scoring rubric for Content/Standards Alignment and Other Factors.

To develop the review instruments, OSPI engaged two groups in two full cycles of development and revision. The IMR Advisory Group and SBE Math Panel were the two primary groups contributing to the development of the instruments. Their work was research based, and used the following primary sources:

- 2008 Washington Revised Math Standards
- NMAP Foundations for Success
- NCTM Curriculum Focal Points

Additionally, the groups also referenced the following secondary sources as resources. Please note that in some instances, the secondary sources were used to compare and contrast effective and ineffective instrument design.

- Math Educators' Summary of Effective Programs
- Park City Mathematics Standards Study Group Report
- Framework for 21st Century Learning
- How People Learn: Brain, Mind, Experience and School
- How Students Learn: Mathematics in the Classroom
- NCTM Principles and Standards for School Mathematics
- Choosing a Standards-Based Mathematics Curriculum – Chapter 6: Developing and Applying Selection Criteria
- Choosing a Standards-Based Mathematics Curriculum – Appendix: Sample Selection Criteria

In addition to seeking advice and guidance from the IMR Advisory Group and the SBE Math Panel, several national and/or external experts were consulted and provided important recommendations for both the process and the review instruments. Several of the external experts provided valuable advice about their state processes where they have successfully completed comprehensive mathematics curriculum reviews.

The outcomes from the review instrument design phase included:

- Two review instruments (Content/Standards Alignment and Other Factors), which are described below
- Proposed threshold and weighting process for final recommendations. Both groups recommended that in order for programs to be considered for the final three recommendations, they must first meet a minimum threshold in content/standards alignment. A scaled score of 0.70 was proposed as this

threshold with a recommendation that the threshold be adjusted if necessary if a sufficient number of materials failed to reach the threshold. In addition, both groups proposed weighting percentages for the Other Factors.

2.1.1 Content/Standards Alignment Threshold

Part 1 of the review measured the alignment of the core/comprehensive instructional materials to the revised 2008 Washington Mathematics standards. Materials that met a minimum threshold of alignment with state standards were considered for inclusion in the list of recommended mathematics curricula.

Reviewers looked for evidence that each Washington state standard Core Process, Content, and Additional Key Information was met in the expected course level.

An additional goal of the Content/Standards Alignment evaluation was to identify the areas where existing materials need supplementation to meet state standards. See *Section 3.1* for charts that show how well each program meets specific Performance Expectations at each course level.

2.1.2 Scale Definitions

Scale	Description
Content/Standards Alignment	The Content/Standards Alignment (Part 1 of the review process) determined to what degree the mathematical concepts, skills and processes were in alignment with revised state mathematics standards. The materials reviewed were accurate, with no errors of fact or interpretation. Adherence to standards implies quality and rigor. It is a fundamental assumption that if the program matches a standard well, the math is accurate, rigorous, and high quality.
Program Organization and Design	Overall program and design. Includes scope and sequence, appropriate use of technology. Content is presented in strands, with definitive beginnings and endings. The program grounds ideas in a bigger framework. The material is logically organized, and includes text-based tools like tables of contents and indexes.
Balance of Student Experience	Tasks lead to the development of core content and process understanding. They present opportunities for students to think about their thinking, develop both skills and understanding, and apply multiple strategies to solve real world problems. Tasks will provide a balance of activities to develop computational fluency and number sense, problem solving skills and conceptual understanding.
Assessment	Tools for teachers and students to formally and informally evaluate learning and guide instruction.

Scale	Description
Instructional Planning and Professional Support	Support for teachers that is embedded in the instructional materials to assist them in teaching the content and standards. Instructional materials provide suggestions for teachers in initiating and orchestrating mathematical discourse. Includes key information about content knowledge to help teachers understand the underlying mathematics. Materials help surface typical student misconceptions and provide ideas for helping address them.
Equity and Access	Support for ELL, unbiased materials, support for gifted and talented students, support for students with disabilities, differentiated instruction, diversity of role models, parent involvement, intervention strategies, quality website, and community involvement ideas.

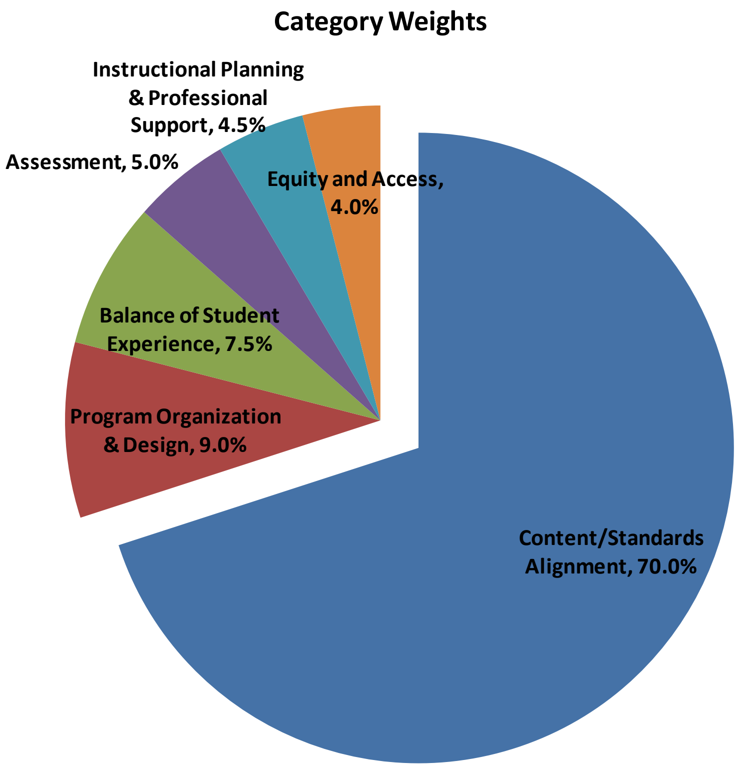

Figure 6. Category weights for the Mathematics Instructional Materials Review. Note that Content/Standards Alignment is both a weighted category and a threshold category, meaning that curricula must meet a minimum average score on content/standards alignment before the material can be considered for possible inclusion in the three recommended core/comprehensive curricula.

Table 12. Measurement scales and weights for/Content Standards Alignment and Other Factors.

Scale	Scale Weight
Content/Standards Alignment	70.0%
Program Organization and Design	9.0%
Balance of Student Experience	7.5%
Assessment	5.0%
Instructional Planning and Professional Support	4.5%
Equity and Access	4.0%

2.1.3 Measurement Criteria

Part 1: Content/Standards Alignment criteria measured how well the Washington state revised mathematics standards were addressed within the materials submitted for review. Reviewers ensured that the mathematics content within the program was rigorous and accurate, with few errors of fact or interpretation. In scoring Part 1, reviewers used a 4 point scale (corresponding with Not Met, Limited Content, Limited Practice, Fully Met) for each performance expectation. This scale uses interval data to represent ordinal data. The criteria are the Washington Revised Mathematics Standards (6/08). A sample rating form for Part 1 is shown below. Note that the raw scores were adjusted to a range of [0, 1] for analysis and display.

Algebra 1		Date:	
Program:		Reviewer #:	

(Rate each item on the scale 0-not met, 1-limited content, 2-limited practice, 3-fully met)							
A1.1. Core Content: Solving problems *(Algebra)*		**0**	**1**	**2**	**3**	**A2**	**Evidence**
A1.1.A	Select and justify functions and equations to model and solve problems.	○	○	○	○	◇	
A1.1.B	Solve problems that can be represented by linear functions, equations, and inequalities.	○	○	○	○	◇	
A1.1.C	Solve problems that can be represented by a system of two linear equations or inequalities.	○	○	○	○	◇	
A1.1.D	Solve problems that can be represented by quadratic functions and equations.	○	○	○	○	◇	
A1.1.E	Solve problems that can be represented by exponential functions and equations.	○	○	○	○	◇	

A1.2. Core Content: Numbers, expressions, and operations *(Numbers, Operations, Algebra)*		**0**	**1**	**2**	**3**	**A2**	**Evidence**
A1.2.A	Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.	○	○	○	○	◇	
A1.2.B	Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.	○	○	○	○	◇	
A1.2.C	Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.	○	○	○	○	◇	
A1.2.D	Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.	○	○	○	○	◇	
A1.2.E	Use algebraic properties to factor and combine like terms in polynomials.	○	○	○	○	◇	
A1.2.F	Add, subtract, multiply, and divide polynomials.	○	○	○	○	◇	

A1.3. Core Content: Characteristics and behaviors of functions *(Algebra)*		**0**	**1**	**2**	**3**	**A2**	**Evidence**
A1.3.A	Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.	○	○	○	○	◇	
A1.3.B	Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.	○	○	○	○	◇	
A1.3.C	Evaluate $f(x)$ at a (i.e., $f(a)$) and solve for x in the equation $f(x) = b$.	○	○	○	○	◇	

A1.4. Core Content: Linear functions, equations, and inequalities *(Algebra)*		**0**	**1**	**2**	**3**	**A2**	**Evidence**
A1.4.A	Write and solve linear equations and inequalities in one variable.	○	○	○	○	◇	
A1.4.B	Write and graph an equation for a line given the slope and the y-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.	○	○	○	○	◇	
A1.4.C	Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.	○	○	○	○	◇	
A1.4.D	Write and solve systems of two linear equations and inequalities in two variables.	○	○	○	○	◇	
A1.4.E	Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.	○	○	○	○	◇	

Figure 7. Sample rating form for Content/Standards Alignment Review.

Reviewers used the following rubric to evaluate and score the Content/Standards Alignment worksheets that were completed by each publisher. During the review week, we posted variance reports that showed the rare instances where two or more independent reviewers had a two point difference on a particular Performance Expectation for a specified program. With clear scoring guidelines this type of variance should not occur; although in the process of collecting 20,000+ data elements some anomalies are expected. In practical terms, if one reviewer selected “Not Met” on a performance expectation for a specific program and another reviewer selected “Fully Met”, there are some possible reasons, including that the initial reviewer might have missed the evidence that shows the performance expectation was fully met. In each case of a variance gap, the discrepancy was highlighted, and reviewers were asked to go back and check their work and/or discuss the differences among each other to understand the reason for the difference. They were given the opportunity to correct their scores or to leave them as-is.

After the review of the K-8 core/comprehensive materials, project leaders sought feedback from participants in that review, the Math Panel, districts, and other stakeholders in order to improve the process for the high school materials review. One key recommendation was to change the content/standards alignment scale to a 4-point scale, with greater differentiation in the middle scores. Below is a table reflecting the updated 4-point scoring rubric.

Table 13. Scoring rubric for Content/Standards Alignment instrument.

There is little or no content (0)	Important content is missing (1)	All or most content is present, but missing some key teaching and learning tools (2)	All content and key teaching and learning tools are present (3)
• All or most of the content in the standard is missing in the program. - It may be completely absent. - It may be briefly mentioned, but it is not developed. - It may contain less sophisticated precursor content that would lead to the content in the standard. • *A typical student would not be able to achieve mastery with the core program materials.*	• Some significant aspect of the content is not present. - Some of the content may be completely absent. - Some of the content may be less rigorous. • It would take significant time and knowledge to fill the content gaps in the program. • *A typical student would not be able to achieve mastery with the core program materials without some content supplementation.*	• The key content from the standard exists in the program. • The core materials need supplementation to do such things as adding additional opportunities for practice or finding other representations to help students consolidate learning. • *Many students would achieve mastery with the core program material.*	• The content from the standard is fully present. • There is adequate information about the content and sufficient teaching and learning ideas included program to ensure that students develop conceptual understanding and procedural skill. • There is sufficient practice to ensure mastery. • *A typical student would be able to achieve mastery with the core program materials.*

We collected additional course level data when the reviewer indicated that the standard was fully met at an alternate course level from the expected level. Algebra 1 and 2 were treated as a series, as well as Integrated Math 1, 2 and 3. Geometry was a standalone course. Reviewers could look at other texts within the series if a particular standard was not addressed in the expected course.

Part 2: Other Factors contributed 30% of the final composite score for each program. There were five scales, with 6-10 elements per scale. In scoring Part 2, reviewers used a consistent, 4-point Likert measurement scale measurement scale for each item (strongly disagree, disagree, agree, strongly agree). A sample instrument form is shown below.

Math Instructional Materials Evaluation – Other Factors

(Rate each item on the scale of 1-Strongly disagree, 2.-Disagree, 3-Agree, 4-Strongly agree)

Grade:		Date:	
Program:		Reviewer#:	

Program Organization and Design		Strongly disagree	disagree	agree	Strongly agree
1.	The content has a coherent and well-developed sequence (organized to promote student learning, links facts and concepts in a way that supports retrieval, builds from & extends concepts previously developed, strongly connects concepts to overarching framework)	O	O	O	O
2.	Program includes a balance of skill-building, conceptual understanding, and application	O	O	O	O
3.	Tasks are varied: some have one correct and verifiable answer; some are of an open nature with multiple solutions	O	O	O	O
4.	The materials help promote classroom discourse	O	O	O	O
5.	The program is organized into units, modules or other structure so that students have sufficient time to develop in-depth major mathematical ideas	O	O	O	O
6.	The instructional materials provide for the use of technology which reflects 21st century ideals for a future-ready student	O	O	O	O
7.	Instructional materials include mathematically accurate and complete indexes and tables of contents to locate specific topics or lessons	O	O	O	O
8.	The materials have pictures that match the text in close proximity, with few unrelated images	O	O	O	O
9.	Materials are concise and balance contextual learning with brevity	O	O	O	O
10.	Content is developed for conceptual understanding: (limited number of key concepts, in-depth development at appropriate age level)	O	O	O	O

Student Learning		1	2	3	4
1.	Tasks lead to conceptual development of core content, procedural fluency, and core processes abilities including solving non-routine problems	O	O	O	O
2.	Tasks build upon prior knowledge	O	O	O	O
3.	Tasks lead to problem solving for abstract, real-world and non-routine problems	O	O	O	O
4.	Tasks encourage students to think about their own thinking	O	O	O	O
5.	The program provides opportunities to develop students' computational fluency using brain power without use of calculators	O	O	O	O
6.	Tasks occasionally use technology to deal with messier numbers or help the students see the math with graphical displays	O	O	O	O
7.	The program promotes understanding and fluency in number sense and operations	O	O	O	O
8.	The program leads students to mastery of rigorous multiple-step word problems	O	O	O	O
9.	The materials develop students' use of standard mathematics terminology/vocabulary	O	O	O	O
10.	Objectives are written for students	O	O	O	O

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Figure 8. Other Factors sample instrument form.

In addition, for each Part 2 category (described above in the Scale Definitions section), stakeholders identified 6-10 criteria, which are shown below.

Program Organization and Design

1. The content has a coherent and well-developed sequence (organized to promote student learning, links facts and concepts in a way that supports retrieval, builds from & extends concepts previously developed, strongly connects concepts to overarching framework)
2. Program includes a balance of skill-building, conceptual understanding, and application
3. Tasks are varied: some have one correct and verifiable answer; some are of an open nature with multiple solutions
4. The materials help promote classroom discourse
5. The program is organized into units, modules or other structure so that students have sufficient time to develop in-depth major mathematical ideas
6. The instructional materials provide for the use of technology with reflects 21st century ideals for a future-ready student

7. Instructional materials include mathematically accurate and complete indexes and tables of contents to locate specific topics or lessons
8. The materials have pictures that match the text in close proximity, with few unrelated images
9. Materials are concise and balance contextual learning with brevity
10. Content is developed for conceptual understanding: (limited number of key concepts, in-depth development at appropriate age level)

Balance of Student Experience

1. Tasks[4] lead to conceptual development of core content, procedural fluency, and core processes abilities including solving non-routine problems
2. Tasks build upon prior knowledge
3. Tasks lead to problem solving for abstract, real-world and non-routine problems
4. Tasks encourage students to think about their own thinking[5]
5. The program provides opportunities to develop students' computational fluency using brain power without use of calculators
6. Tasks occasionally use technology to deal with messier numbers or help the students see the math with graphical displays
7. The program promotes understanding and fluency in number sense and operations
8. The program leads students to mastery of rigorous multiple-step word problems
9. The materials develop students' use of standard mathematics terminology/vocabulary
10. Objectives are written for students

Instructional Planning and Professional Support

1. The instructional materials provide suggestions to teachers on how to help students access prior learning as a foundation for further math learning
2. The instructional materials provide suggestions to teachers on how to help students learn to conjecture, reason, generalize and solve problems
3. The instructional materials provide suggestions to teachers on how to help students connect mathematics ideas and applications to other math topics, other disciplines and real world context
4. Background mathematics information is included so that the concept is explicit in the teacher guide
5. Instructional materials help teachers anticipate and surface common student misconceptions in the moment
6. The materials support a balanced methodology
7. Math concepts are addressed in a context-rich setting (giving examples in context, for instance)
8. Teacher's guides are clear and concise with easy to understand instructions

Assessment

---

[4] Tasks can include homework, lessons, in-class group or individual activities, assessments, etc.
[5] Students are expected to be able to analyze their thinking process to understand how they came to a conclusion.

1. The program provides regular assessments to guide student learning
2. There are opportunities for student self-assessment of learning
3. Assessments reflect content, procedural, and process goals and objectives
4. The program includes assessments with multiple purposes (formative, summative and diagnostic)
5. Assessments include multiple choice, short answer and extended response formats.
6. Recommended rubrics or scoring guidelines accurately reflect learning objectives
7. Recommended rubrics or scoring guidelines identify possible student responses both correct & incorrect
8. Accurate answer keys are provided

Equity and Access

1. The program provides methods and materials for differentiating instruction (students with disabilities, gifted/talented, English Language Learners (ELL), disadvantaged)
2. Materials support intervention strategies
3. Materials, including assessments are unbiased and relevant to diverse cultures
4. Materials are available in a variety of languages
5. The program includes easily accessible materials which help families to become active participants in their students' math education (e.g. "How You Can Help at Home" letters with explanations, key ideas & vocabulary for each unit, free or inexpensive activities which can be done at home, ideas for community involvement[6])
6. The program includes guidance and examples to allow students with little home support to be self-sufficient and successful

2.2 Reviewer Selection Process

OSPI issued a statewide invitation to solicit applications from individuals interested in serving as mathematics Professional Development Facilitators (trainers on the revised standards) and/or to participate as Instructional Materials Reviewers Committee members. Over 400 applications were received for both roles. Using a common review instrument and criteria, a committee reviewed and scored the over 100 applications for the instructional materials review and selected 42 individuals. The IMR Committee was selected first based on the score of their application (primarily based on experience). Next, it was important to have a balanced number of reviewers qualified to review algebra, geometry and integrated math levels. In addition, OSPI sought balance on the review team, ensuring that math educators, curriculum specialists, parents, advocacy group members, mathematicians and math coaches were represented in the final group. Parent recommendations were solicited from the Washington State Parent Teacher Association and Where's the Math.

[6] Community involvement means ideas where students can apply math concepts they are learning in the context of business, environment or public service for example.

2.3 Publisher Involvement

All publishers were invited to submit core/comprehensive mathematics material for review. The materials did not have to be in widespread use in Washington in order to be considered. Information about the review was disseminated widely by the Washington Oregon Alaska Textbook Representatives Association (WOATRA), the American Association of Publishers (AAP) and available on the OSPI Publisher Notice web site.

In addition, OSPI hosted a Publisher's Meeting to address questions prior to the review. As a result, OSPI maintained a web-based Question and Answer document for the publishers, so they had up-to-date information regarding the submission and review process.

In addition to providing curricular materials for review, publishers were asked to review their materials and compare them to the 2008 WA Revised Mathematics Standards. For each program submitted for review, publishers completed a Program Alignment Worksheet that provided between one and five references to locations in their materials where the standard was presented.

Publishers also submitted a Professional Development plan that outlined what standard professional development was available with the purchase of materials, and the optimal, recommended amount and type of professional development.

Publishers delivered materials to the review site the day before the review. They were escorted into the library repository, and participated in an inventory check with OSPI staff. After the review week was completed, they collected their material. Publishers did not meet with or present to the IMR Review Committee.

2.4 Review Week Process

The high school core/comprehensive mathematics review week took place in SeaTac, Washington from November 9-14, 2008.

On Sunday, November 9, the review team participated in an eight-hour mathematics standards training, led by Dr. George Bright from OSPI. The purpose of this training was to familiarize the reviewers with the standards at the course levels they would be reviewing. Dr. Bright provided clarity on the meaning of each standard, and example evidence that shows how the standard could be developed in instructional materials.

Reviewers participated in another four-hour training on Monday morning that focused on the review instruments (Content/Standards Alignment and Other Factors), how to score the elements, and expectations for reviewers, such as independent assessments, bias-free professional judgments, consistent scoring and productivity expectations.

Between Monday afternoon and Friday morning, reviewers read and evaluated all materials submitted. They checked out programs (and ancillary materials, if submitted) from the library, and spent on average about 3.5 hours per program-grade evaluating and scoring the material.

Staff entered data from the instruments in near real-time. Twice per day, the group gathered for progress updates, variance checks and process improvement changes.

The initial expectation was for each program-grade to receive three independent reviews. However, the reviewers ended up working both before and after the standard day (The review room was open between 6 a.m. and 9 p.m. daily) and were able to complete four reviews per program-grade for all of the instructional materials reviewed.

2.5 Data Analysis Process/Methodology

The purpose of this section is to describe in easy to understand terms how the data were analyzed. For example, it describes the process by which programs met a threshold level and how the comprehensive score is calculated (with weights).

Professional data entry staff entered the data into an Access database in near real-time. Once the review week was complete, we extracted the scores into a flat-file Excel worksheet for graphics publication and also text file format for statistical processing using the statistical package R.

Two statisticians worked independently with the data, first doing exploratory data analysis, looking for any anomalies or outliers (like a score value of 11, when the max score value should have been a 1). The statisticians checked counts of data, ranges, distributions and variance, as examples. No entry or extract errors were apparent, which was expected given the input constraints on the data entry application.

Some data cleaning and recoding ensued. Several program names were shortened or clarified to prepare the data for final graphic presentation.

The data for the Other Factors scales had an original range of [1,4] and the Content/Standards Alignment scale had a range of [0,3]. Before scaling the data and converting it to a common [0,1] range, the Other Factors range was adjusted to [0,3]. This was done to prevent an inflation of the Other Factors after the data was adjusted. (If a range of [1,4] is divided by 4, it becomes [0.25,1], which cannot be directly compared to the scaled content score at [0,1].)

After exploratory data analysis (EDA) and the data cleaning/recoding were completed, we re-checked the accuracy of the data elements by randomly sampling 10% of the original data entry forms and comparing them to the values in the electronic data set. Only 0.06% of items on the sampled forms were found to be entered incorrectly (and corrected), indicating a high level of accuracy in the data entry.

The final composite score was calculated by multiplying the scaled average values by the scale weights and summing the values. Confidence intervals were set at 95% and calculated for each instructional materials series.

One important consideration in ranking the data is to identify where statistical ties might occur. The tables and graphs that show confidence intervals for each instructional materials series are critical for understanding that small differences in composite scores may be due to sampling or other error (including measurement error) rather than a true difference in means.

The most critical statistical tie in the ranked list of composite scores involves the recommended programs and subsequent lower ranked instructional materials series. For example, if the third, fourth and fifth ranked series are statistical ties, then the simple ranking is not sufficient justification alone to select and recommend the set of the first through third ranked instructional materials.

To test for statistical ties, we used a one-tailed t-test and accounted for multiple tests. Prior to collecting the data, the statistical team considered several statistical tests, and decided to use the one-tailed t-test for three reasons: 1) the expected number of data elements, the expected distribution of the averages and the data type (ordinal converted to interval) made the t-test a good fit; 2) the t-test is one of the most commonly used and most easily understood statistical tests to use; and 3) it provides a very robust mechanism for measuring differences of means.

We want to identify any statistical ties with the recommended curricula in each course type. To do so, it is sufficient to ascertain if any curriculum has a statistically equivalent rating to the last rated program in the set of recommendations. The following example assumes the selection of the top three ranked programs, and a comparison of the third-ranked program to lower ranked (4^{th}, 5^{th}, etc.) programs.

First, we perform hypothesis tests comparing the ratings of all lower ranked materials to the third.

H_O: rating 3 = rating [4…n]
H_A: rating 3 > rating [4…n]

The test is a one-sided two-sample t-test. To allow for differences in the variances of the means across materials, we used an unequal variance statistic:

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}}$$

Where the standard error of the difference is calculated by:

$$s_{\overline{x_1}-\overline{x_2}} = \sqrt{se_1^2 + se_2^2}$$

See *Section 5.9* for the degrees of freedom calculations for the following tables.

Table 14 and Table 15 give the adjusted significance levels for algebra, geometry and integrated math respectively, calculated using the Holm-Bonferroni method. Since we are performing several comparisons for each course type, we need to correct for multiple testing. Rather than comparing each p-value to 0.05, we order the p-values from smallest to largest and then compare them, in order, to the nominal significance level (0.05) divided by the number of tests remaining. When we reach a p-value that is deemed insignificant, we then say that all remaining values are also insignificant.

Table 14. t-test results comparing lower-scoring programs to the third-highest scoring Algebra 1 and 2 series.

Program	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Discovering - Algebra	0.859					
Holt Algebra	0.832					
Glencoe McGraw-Hill Algebra	0.821					
MathConnections Algebra	0.532	-21.08	98	2.69E-38	9	0.006
PH Classics (Smith) Algebra	0.658	-12.28	90	3.11E-21	8	0.006
PH Classics (Foerster) Algebra	0.672	-10.48	93	1.14E-17	7	0.007
CORD Algebra	0.699	-8.71	88	8.88E-14	6	0.008
Cognitive Tutor Algebra	0.714	-8.47	89	2.49E-13	5	0.010
CME Algebra	0.731	-6.47	95	2.10E-09	4	0.013
McDougal Littell Algebra	0.752	-5.31	89	4.05E-07	3	0.017
CPM Algebra	0.768	-3.63	94	2.31E-04	2	0.025
PH Math Algebra	**0.814**	**-0.59**	**86**	**0.277**	**1**	**0.050**

Table 15. t-test results comparing lower-scoring programs to the third-highest scoring geometry program.

Program	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Holt Geometry	0.847					
McDougal Littell Geometry	0.843					
Glencoe McGraw-Hill Geometry	0.832					
MathConnections Geometry	0.528	-17.33	73	1.07E-27	7	0.007
CME Geometry	0.613	-12.79	76	7.78E-21	6	0.008
CPM Geometry	0.729	-6.41	83	4.35E-09	5	0.010
Cognitive Tutor Geometry	0.730	-5.61	70	1.95E-07	4	0.013
Discovering - Geometry	0.776	-3.25	76	8.63E-04	3	0.017
CORD Geometry	0.795	-2.21	80	0.015	2	0.025
PH Math Geometry	**0.827**	**-0.31**	**87**	**0.377**	**1**	**0.050**

Prentice Hall Math Algebra is the fourth-ranked algebra series. It is not statistically different from the third-ranked program, Glencoe McGraw-Hill Algebra.

Of the geometry programs, only Prentice Hall Geometry is not statistically different from the third-ranked program, Glencoe McGraw-Hill Geometry. However, all remaining curricula are significantly different from the third-highest rated program.

Only Core Plus Mathematics and SIMMS Math exceeded the content/standards alignment threshold for the integrated programs, when treated as a series. The second and third ranked integrated program mean scores are statistically different from Core Plus Mathematics.

Table 16. t-test results comparing integrated programs.

Program	Mean score	t statistic	Degrees of freedom	p-value
Core Plus Math	0.780			
SIMMS Math	0.696	-6.78	112	3.05E-10
Interactive Math Program	0.621	-11.96	102	2.35E-21

3 Results

3.1 Content/Standards Alignment

The following graphs show ranked results for the content/standards alignment scale for all the series that were reviewed (Algebra 1 and 2, Geometry, and Integrated Math 1, 2 and 3)

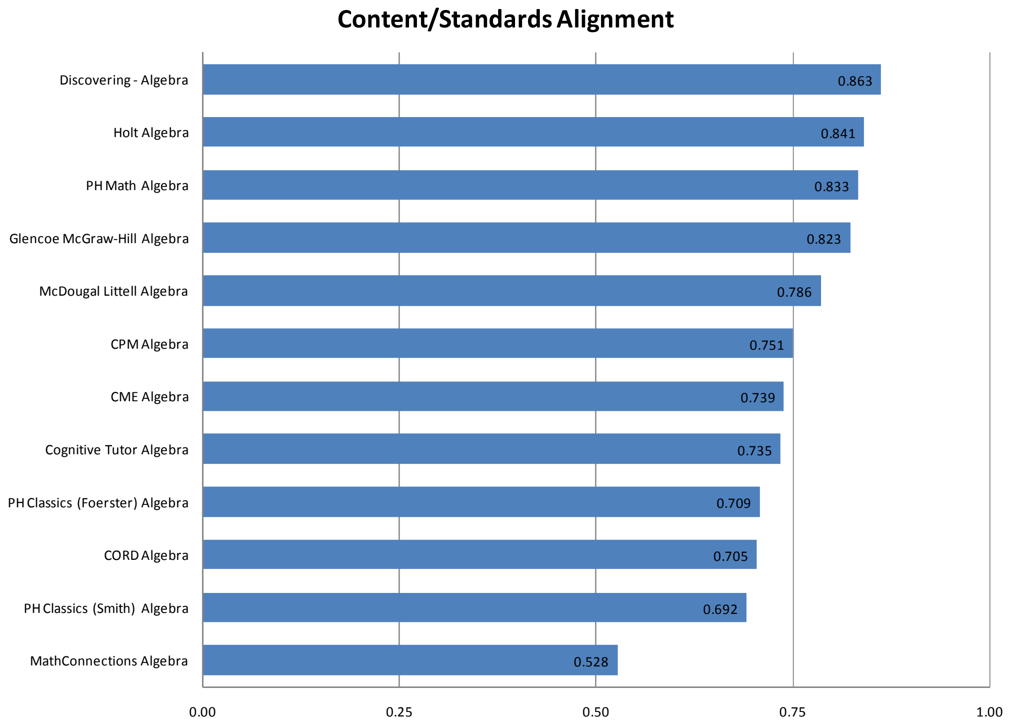


Figure 9. Algebra 1 and 2 series content/standards alignment scale, in rank order.

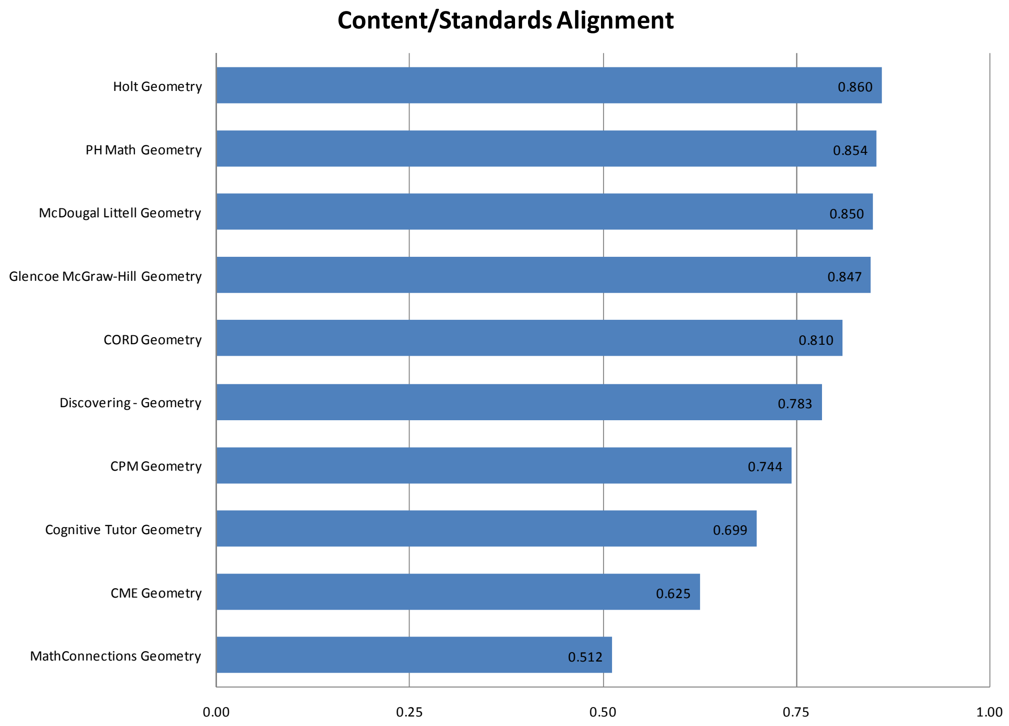


Figure 10. Geometry programs content/standards alignment scale.

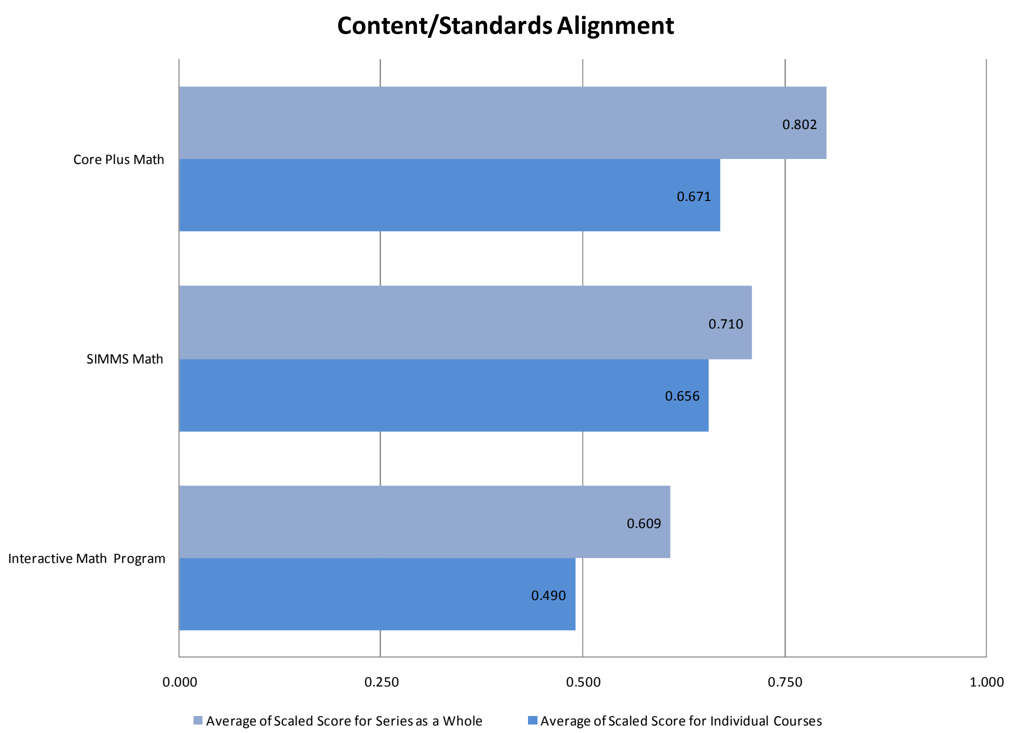


Figure 11. Integrated programs content/standards alignment scale, treated as a series (light blue) and as individual courses (dark blue).

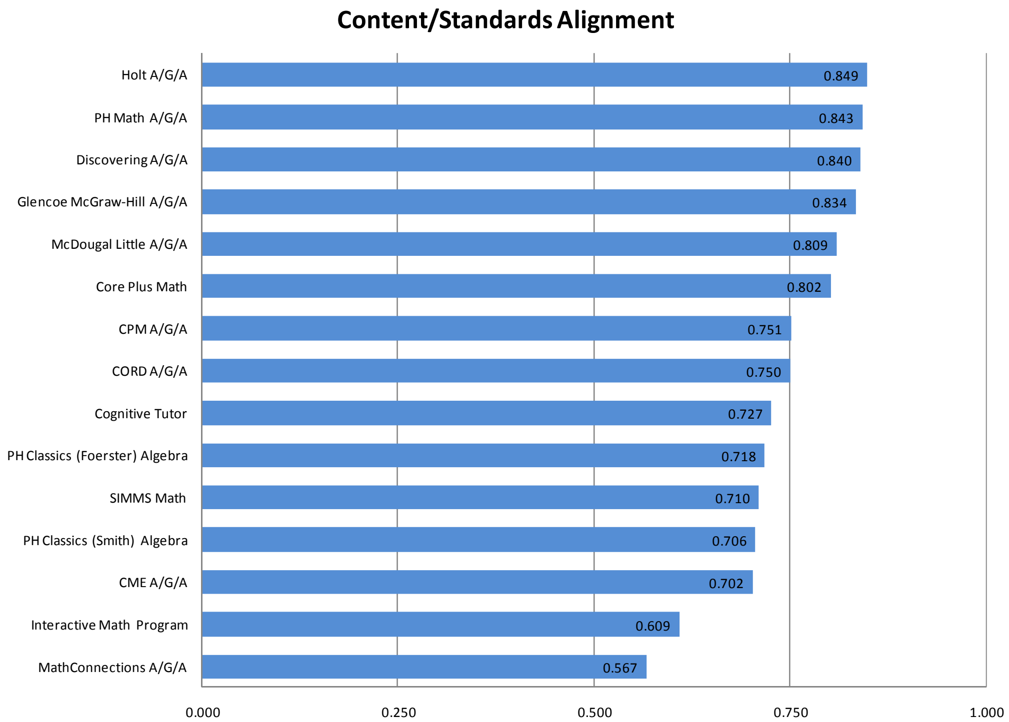


Figure 12. Content/Standards Alignment for all publisher bundles, treated as a series (no reduction in score for standards met above or below the expected course level).

3.2 Content Dashboards

The following tables show summary and detailed information about content. The dashboard view shows a filled circle if the scaled average score from the reviewers is ≥ 0.70 (on a 1.0 scale); a half circle if the scale is between 0.50 and 0.69 inclusive, and a clear circle if the average score is below 0.50.

3.2.1 Summary

Table 17. Core Content Area summary for Algebra 1 courses.

Core Content Area	Glencoe McGraw-Hill Algebra	Discovering - Algebra	PH Math Algebra	Holt Algebra	CPM Algebra	McDougal Littell Algebra	CORD Algebra	CME Algebra	Cognitive Tutor Algebra	PH Classics (Smith) Algebra	PH Classics (Foerster) Algebra	MathConnections Algebra	Overall
Solving Problems	●	●	●	●	●	●	●	●	●	●	●	○	●
Numbers, expressions and operations	●	◑	●	●	●	●	◑	●	●	●	●	○	●
Characteristics and behaviors of functions	●	●	●	●	●	●	●	●	●	◑	●	○	●
Linear functions, equations and inequalities	●	●	●	●	●	●	●	●	●	●	●	○	●
Quadratic functions and equations	●	●	●	●	●	●	●	●	●	●	●	○	●
Data and distributions	●	◑	◑	◑	○	◑	◑	◑	◑	◑	○	◑	◑
Additional Key Content	●	●	●	●	◑	●	◑	◑	◑	○	○	○	◑
Reasoning, Problem Solving, and Communication	●	●	●	●	●	●	●	●	●	◑	◑	◑	●
Overall	●	●	●	●	●	●	●	●	●	◑	◑	○	●

Table 18. Core Content Area summary for Geometry courses.

Core Content Area	Holt Geometry	PH Math Geometry	McDougal Littell Geometry	Glencoe McGraw-Hill Geometry	CORD Geometry	Discovering - Geometry	CPM Geometry	Cognitive Tutor Geometry	CME Geometry	MathConnections Geometry	Overall
Logical arguments and proofs	●	●	●	●	●	●	◑	○	○	○	●
Lines and angles	●	●	●	●	●	●	●	●	◑	◑	●
Two- and Three-Dimensional Figures	●	●	●	●	●	●	●	●	●	◑	●
Geometry in the coordinate plane	●	●	●	●	●	●	◑	◑	◑	○	◑
Geometric transformations	●	●	●	●	●	●	●	●	◑	○	●
Additional Key Content	●	◑	◑	●	○	◑	◑	○	○	◑	◑
Reasoning, Problem Solving, and Communication	●	●	●	●	●	●	●	●	●	●	●
Overall	●	●	●	●	●	●	●	◑	◑	◑	●

Table 19. Core Content Area summary for Algebra 2 courses.

Core Content Area	Discovering - Algebra	Holt Algebra	PH Math Algebra	Glencoe McGraw-Hill Algebra	PH Classics (Foerster) Algebra	McDougal Littell Algebra	Cognitive Tutor Algebra	PH Classics (Smith) Algebra	CPM Algebra	CME Algebra	MathConnections Algebra	CORD Algebra	Overall
Solving Problems	●	●	●	●	●	●	●	●	◑	◑	◑	●	●
Numbers, expressions and operations	●	●	●	●	●	●	●	●	◑	●	○	◑	●
Quadratic functions and equations	●	●	●	●	●	●	●	●	◑	◑	○	◑	●
Exponential and logarithmic functions and equations	●	●	●	●	●	●	●	●	●	●	●	●	●
Additional functions and equations	●	●	●	●	◑	●	●	○	●	●	○	○	●
Probability, data, and distributions	●	●	●	●	◑	●	◑	◑	◑	○	●	○	◑
Additional Key Content	●	●	●	●	●	●	◑	●	●	●	◑	●	●
Reasoning, Problem Solving, and Communication	●	●	●	●	●	◑	◑	◑	◑	●	●	◑	●
Overall	●	●	●	●	●	●	●	●	●	◑	◑	◑	●

Table 20. Core Content Area summary for Integrated Math 1 courses, treated as a series (no reductions in score for standards met above or below the expected course level).

Core Content Area	Core Plus Math	SIMMS Math	Interactive Math Program	Overall
Solving Problems	●	●	◑	●
Numbers, expressions and operations	●	○	◑	◑
Characteristics and behaviors of functions	●	●	◑	●
Linear functions, equations and relationships	●	●	○	◑
Proportionality, similarity, and geometric reasoning	●	○	◑	◑
Data and distributions	●	◑	●	●
Additional Key Content	●	◑	○	◑
Reasoning, Problem Solving, and Communication	●	●	●	●
Overall	●	◑	◑	●

Table 21. Core Content Area summary for Integrated Math 2 courses, treated as a whole series.

Core Content Area	Core Plus Math	SIMMS Math	Interactive Math Program	Overall
Modeling situations and solving problems	●	●	●	●
Quadratic functions, equations, and relationships	●	○	◑	◑
Conjectures and proofs	●	●	○	◑
Probability	●	●	●	●
Additional Key Content	●	◑	◑	◑
Reasoning, Problem Solving, and Communication	●	●	●	●
Overall	●	●	◑	●

Table 22. Core Content Area summary for Integrated Math 3, treated as a whole series.

Core Content Area	SIMMS Math	Core Plus Math	Interactive Math Program	Overall
Solving Problems	●	●	◑	●
Transformations and functions	●	●	○	◑
Functions and modeling	◑	◑	○	◑
Quantifying variability	●	◑	○	◑
Three-dimensional geometry	●	○	○	◑
Algebraic properties	○	●	○	○
Additional Key Content	◑	◑	○	◑
Reasoning, Problem Solving, and Communication	●	●	●	●
Overall	●	●	○	◑

Table 23. Core Content Area summary for Integrated Math 1, treated as an individual course (score reductions applied when standard is met above or below the expected course level.)

Core Content Area	Core Plus Math	SIMMS Math	Interactive Math Program	Overall
Solving Problems	●	●	◑	●
Numbers, expressions and operations	◑	○	◑	◑
Characteristics and behaviors of functions	○	●	○	◑
Linear functions, equations and relationships	●	●	○	◑
Proportionality, similarity, and geometric reasoning	◑	○	◑	◑
Data and distributions	●	◑	●	●
Additional Key Content	●	◑	○	◑
Reasoning, Problem Solving, and Communication	●	●	●	●
Overall	●	◑	◑	◑

Table 24. Core Content Area summary for Integrated Math 2, treated as an individual course.

Core Content Area	SIMMS Math	Core Plus Math	Interactive Math Program	Overall
Modeling situations and solving problems	●	●	◑	●
Quadratic functions, equations, and relationships	○	◑	○	○
Conjectures and proofs	◑	○	○	○
Probability	●	◑	○	◑
Additional Key Content	○	◑	○	○
Reasoning, Problem Solving, and Communication	●	●	●	●
Overall	◑	◑	○	◑

Table 25. Core Content Area summary for Integrated Math 3, treated as an individual course.

Core Content Area	SIMMS Math	Core Plus Math	Interactive Math Program	Overall
Solving Problems	◑	●	◑	●
Transformations and functions	●	○	○	○
Functions and modeling	◑	◑	○	○
Quantifying variability	●	◑	○	○
Three-dimensional geometry	◑	○	○	○
Algebraic properties	○	◑	○	○
Additional Key Content	○	◑	○	○
Reasoning, Problem Solving, and Communication	●	●	●	●
Overall	◑	◑	○	◑

3.2.2 Detail

Table 26 shows the degree in which the Algebra 1 and 2 materials reviewed meet each Performance Expectation for Algebra 1. The dashboard view shows a filled circle if the scaled average score from the four reviewers is ≥ 0.70 (on a 1.0 scale); a half circle if the scale is between 0.50 and 0.69 inclusive, and a clear circle if the average score is below 0.50.

The programs are listed in rank order from left to right based on the average score across all Algebra 1 performance expectations. For example, *Glencoe McGraw-Hill Algebra*, with an overall average Algebra 1 rating on content/standards alignment of 0.82 is shown first.

There are a couple of key conjectures that could be drawn from this chart. The standards are organized into sections or core content areas, (A1.1.A through A1.1.D for example). Some programs are very strong in some sections while weak across other sections. See for instance, *CPM Algebra*, which performs well in *A1.1 Solving Problems, A1.2 Numbers, expressions and operations, A1.3 Characteristics and behaviors of functions, A1.4 Linear functions, equations and inequalities, A1.5Quadratic functions and equations, A1.7 Additional Key Content,* and *A1.8 Reasoning, Problem Solving, and Communication*, but is very weak in *A1.6 Data and distributions*. Thus, it may be that certain instructional materials need to be heavily supplemented in some key content areas. It might also be noted that some areas are easier to supplement than others. For example, given the large volume of computational fluency programs available, it might be easier to supplement numbers and operations than reasoning and problem solving.

Additionally, the far right column shows how all programs performed overall for each specific performance expectation. For example, standard A1.8.A (*Analyze a problem* *situation and represent it mathematically*) is well covered in all reviewed programs, but standard A1.6.C (*Describe how linear transformations affect the center and spread of univariate data)* is not well covered in any program. This data may provide valuable feedback in understanding which of the revised math standards may need supplementation to support a majority of the students in the state.

Table 26. Performance Expectation Dashboard for Algebra 1 courses.

PE	Glencoe McGraw-Hill Algebra	Discovering - Algebra	PH Math Algebra	Holt Algebra	CPM Algebra	McDougal Littell Algebra	CORD Algebra	CME Algebra	Cognitive Tutor Algebra	PH Classics (Smith) Algebra	PH Classics (Foerster) Algebra	MathConnections Algebra	Overall
Solving Problems	●	●	●	●	●	●	●	●	●	●	●	○	●
A1.1.A	●	●	●	●	●	●	●	●	●	◑	●	◑	●
A1.1.B	●	●	●	●	●	●	●	●	●	●	●	◑	●
A1.1.C	●	●	●	●	●	●	●	●	●	●	●	◑	●
A1.1.D	●	●	●	●	●	●	●	●	●	●	●	○	●
A1.1.E	●	●	●	●	◑	●	◑	●	●	○	◑	◑	◑
Numbers, expressions and operations	●	◑	●	●	●	●	◑	●	●	●	●	○	●
A1.2.A	●	○	●	◑	●	●	◑	●	●	●	●	○	●
A1.2.B	●	●	●	●	●	●	●	●	◑	●	●	◑	●
A1.2.C	●	●	●	●	●	●	●	●	◑	●	●	○	●
A1.2.D	○	◑	○	○	○	◑	○	○	◑	○	◑	○	○
A1.2.E	●	●	●	●	●	●	●	●	●	●	●	○	●
A1.2.F	●	○	●	●	●	●	◑	◑	●	●	●	○	●
Characteristics and behaviors of functions	●	●	●	●	●	●	●	●	●	◑	●	◑	●
A1.3.A	●	●	●	●	●	●	●	◑	◑	◑	●	○	●
A1.3.B	●	●	●	●	●	●	●	●	●	●	●	◑	●
A1.3.C	●	●	●	◑	●	●	◑	●	●	◑	◑	○	●
Linear functions, equations and inequalities	●	●	●	●	●	●	●	●	●	●	●	○	●
A1.4.A	●	●	●	●	●	●	●	●	●	●	●	○	●
A1.4.B	●	●	●	●	●	●	●	●	●	●	●	◑	●
A1.4.C	●	●	●	●	●	●	●	◑	●	●	◑	◑	●
A1.4.D	●	●	●	●	●	●	●	●	●	●	●	○	●
A1.4.E	◑	●	●	●	●	●	●	●	●	○	○	○	◑
Quadratic functions and equations	●	●	●	●	●	●	●	●	●	●	●	○	●
A1.5.A	●	●	◑	●	●	●	●	●	●	●	●	○	●
A1.5.B	●	●	●	●	●	●	●	●	●	●	●	○	●
A1.5.C	●	●	●	●	●	●	●	●	◑	●	●	○	●
A1.5.D	●	●	●	●	●	●	●	●	●	●	●	○	●
Data and distributions	●	◑	◑	◑	○	◑	◑	◑	◑	◑	○	◑	◑
A1.6.A	●	●	◑	●	○	●	◑	●	●	◑	○	●	◑
A1.6.B	●	●	◑	◑	○	●	◑	●	◑	○	○	●	◑
A1.6.C	○	○	○	○	○	○	○	○	○	○	○	○	○
A1.6.D	●	●	●	◑	◑	●	●	●	◑	◑	◑	●	●
A1.6.E	●	○	●	●	◑	●	●	◑	◑	●	○	○	◑
Additional Key Content	●	●	●	●	◑	●	◑	◑	◑	○	○	○	◑
A1.7.A	◑	◑	●	●	◑	●	◑	◑	◑	○	○	○	◑
A1.7.B	●	●	●	●	○	◑	●	●	●	○	○	○	◑
A1.7.C	◑	●	●	◑	●	◑	◑	◑	○	○	○	○	○
A1.7.D	●	●	◑	●	○	●	◑	◑	●	●	●	○	◑
Reasoning, Problem Solving, and Communication	●	●	●	●	●	●	●	●	●	◑	◑	◑	●
A1.8.A	●	●	●	●	●	●	●	●	●	●	●	●	●
A1.8.B	●	●	●	●	●	●	●	●	◑	◑	●	○	●
A1.8.C	●	●	●	◑	●	●	●	●	●	◑	○	●	●
A1.8.D	●	●	●	◑	●	●	●	●	●	○	◑	◑	●
A1.8.E	●	●	●	●	●	●	●	●	◑	◑	◑	●	●
A1.8.F	●	◑	◑	●	●	◑	●	●	●	◑	○	◑	●
A1.8.G	◑	●	◑	◑	●	◑	●	●	◑	○	○	●	◑
A1.8.H	●	●	◑	●	●	◑	○	●	○	◑	○	○	◑
Overall	●	●	●	●	●	●	●	●	●	◑	◑	○	●

Table 27. Performance Expectation Dashboard for Geometry.

PE	Holt Geometry	PH Math Geometry	McDougal Littell Geometry	Glencoe McGraw-Hill Geometry	CORD Geometry	Discovering - Geometry	CPM Geometry	Cognitive Tutor Geometry	CME Geometry	MathConnections Geometry	Overall
Logical arguments and proofs	●	●	●	●	●	●	◑	○	○	○	●
G.1.A	●	●	●	●	●	●	◑	○	○	○	◑
G.1.B	●	●	●	●	●	●	●	◑	○	●	●
G.1.C	●	●	●	●	●	●	●	●	●	◑	●
G.1.D	●	●	●	●	●	●	◑	○	○	○	◑
G.1.E	●	●	●	●	●	●	◑	○	●	◑	●
G.1.F	●	●	●	●	◑	●	◑	○	◑	○	◑
Lines and angles	●	●	●	●	●	●	●	●	◑	◑	●
G.2.A	●	●	●	●	●	●	◑	●	●	○	●
G.2.B	●	●	●	●	●	●	◑	●	◑	◑	●
G.2.C	●	●	◑	●	●	●	●	◑	◑	◑	●
G.2.D	●	●	●	●	◑	◑	◑	◑	○	◑	◑
Two- and Three-Dimensional Figures	●	●	●	●	●	●	●	●	●	◑	●
G.3.A	●	●	●	●	●	●	●	●	◑	●	●
G.3.B	●	●	●	●	●	●	●	●	●	◑	●
G.3.C	●	●	●	●	●	●	●	●	◑	○	●
G.3.D	●	●	●	●	●	●	◑	●	●	●	●
G.3.E	●	●	●	●	●	●	●	●	●	●	●
G.3.F	●	●	●	●	●	●	●	●	◑	◑	●
G.3.G	●	●	●	●	●	●	●	●	◑	◑	●
G.3.H	●	●	●	●	●	●	●	●	●	○	●
G.3.I	●	●	◑	●	○	●	◑	●	◑	◑	◑
G.3.J	◑	●	●	●	○	●	◑	●	◑	●	◑
G.3.K	●	●	●	●	●	◑	◑	●	●	◑	●
Geometry in the coordinate plane	●	●	●	●	●	●	◑	◑	◑	○	◑
G.4.A	●	●	●	●	●	●	○	●	◑	○	●
G.4.B	●	●	●	●	●	●	◑	●	●	○	●
G.4.C	●	●	◑	●	●	◑	●	●	○	○	◑
G.4.D	●	●	●	◑	◑	○	◑	○	◑	○	◑
Geometric transformations	●	●	●	●	●	●	●	●	◑	○	●
G.5.A	●	●	●	●	●	●	●	●	◑	○	●
G.5.B	●	●	●	●	●	●	●	●	●	○	●
G.5.C	◑	◑	●	◑	●	●	◑	●	◑	○	◑
G.5.D	●	●	●	●	●	●	●	◑	●	○	●
Additional Key Content	●	◑	◑	●	○	◑	◑	○	○	◑	◑
G.6.A	●	●	●	●	●	●	●	●	◑	◑	●
G.6.B	◑	○	◑	●	○	○	◑	○	○	○	○
G.6.C	●	●	●	●	●	●	●	●	●	◑	●
G.6.D	●	●	◑	●	◑	◑	●	◑	●	○	●
G.6.E	◔	○	◑	◑	○	◑	○	○	◑	○	○
G.6.F	○	○	○	●	○	○	○	○	○	◑	○
Reasoning, Problem Solving, and Communication	●	●	●	●	●	●	●	●	●	●	●
G.7.A	●	●	●	●	●	●	●	●	●	●	●
G.7.B	●	●	●	●	●	●	●	●	●	●	●
G.7.C	●	●	●	●	●	◑	●	●	●	◑	●
G.7.D	●	●	●	●	●	●	●	●	●	◑	●
G.7.E	●	●	●	●	●	●	●	●	●	◑	●
G.7.F	●	●	●	●	●	◑	●	●	◑	●	●
G.7.G	●	●	●	●	●	●	●	●	●	●	●
G.7.H	●	●	●	●	●	●	●	●	◑	◑	●
Overall	●	●	●	●	●	●	●	◑	◑	◑	●

Table 28. Performance Expectation Dashboard for Algebra 2.

PE	Discovering - Algebra	Holt Algebra	PH Math Algebra	Glencoe McGraw-Hill Algebra	PH Classics (Foerster) Algebra	McDougal Littell Algebra	Cognitive Tutor Algebra	PH Classics (Smith) Algebra	CPM Algebra	CME Algebra	MathConnections Algebra	CORD Algebra	Overall
Solving Problems	●	●	●	●	●	●	●	●	◑	◑	◑	●	●
A2.1.A	●	●	●	◑	●	◑	●	◑	●	●	○	◑	●
A2.1.B	●	●	●	●	●	●	●	●	●	●	○	●	●
A2.1.C	●	●	●	●	◑	●	◑	●	◑	○	◑	●	●
A2.1.D	●	●	●	●	●	●	●	●	●	●	●	●	●
A2.1.E	●	●	●	◑	●	◑	◑	●	○	○	○	○	◑
A2.1.F	●	●	●	○	●	●	●	●	●	○	●	●	●
Numbers, expressions and operations	●	●	●	●	●	●	●	●	◑	●	○	◑	●
A2.2.A	●	●	●	○	●	◑	◑	●	◑	◑	◑	○	◑
A2.2.B	●	●	●	●	●	●	●	●	◑	●	◑	○	●
A2.2.C	●	●	●	●	●	●	●	●	◑	●	○	●	●
Quadratic functions and equations	●	●	●	●	●	●	●	●	◑	◑	○	◑	●
A2.3.A	●	●	●	●	●	●	●	●	●	◑	○	○	●
A2.3.B	●	●	●	●	●	●	●	●	○	●	◑	●	●
A2.3.C	●	●	●	●	◑	●	◑	◑	◑	○	○	◑	◑
Exponential and logarithmic functions and equations	●	●	●	●	●	●	●	●	●	●	●	●	●
A2.4.A	●	●	●	●	●	●	●	●	●	●	●	●	●
A2.4.B	●	●	●	●	●	●	●	●	●	●	●	●	●
A2.4.C	●	●	●	●	●	●	●	●	●	●	◑	●	●
Additional functions and equations	●	●	●	●	◑	●	●	○	●	●	○	○	●
A2.5.A	●	●	●	●	○	●	●	●	●	●	◑	●	●
A2.5.B	●	●	●	●	◑	●	●	○	●	●	○	○	◑
A2.5.C	●	●	●	●	●	●	●	○	◑	◑	◑	○	◑
A2.5.D	●	●	●	●	●	●	●	◑	◑	●	◑	○	●
Probability, data, and distributions	●	●	●	●	◑	●	◑	◑	◑	○	●	○	◑
A2.6.A	●	●	●	◑	●	●	●	●	●	◑	●	●	●
A2.6.B	●	●	●	●	●	●	◑	●	●	◑	●	●	●
A2.6.C	●	●	●	●	●	●	●	●	●	○	●	●	●
A2.6.D	●	●	●	●	●	●	◑	◑	●	○	◑	○	●
A2.6.E	●	●	○	●	◑	●	●	○	○	○	○	○	◑
A2.6.F	●	◑	●	●	◑	◑	◑	●	●	○	●	○	◑
A2.6.G	●	○	●	●	○	◑	○	○	○	○	◑	○	○
Additional Key Content	●	●	●	●	●	●	◑	●	●	●	◑	●	●
A2.7.A	●	●	●	●	●	●	●	●	●	●	○	●	●
A2.7.B	●	●	●	●	●	●	◑	●	●	●	◑	●	●
Reasoning, Problem Solving, and Communication	●	●	●	●	●	◑	◑	◑	◑	●	●	◑	●
A2.8.A	●	●	●	●	●	●	●	●	●	●	●	●	●
A2.8.B	●	●	◑	◑	●	◑	◑	●	●	●	◑	◑	●
A2.8.C	●	●	●	●	●	○	●	◑	◑	◑	●	●	●
A2.8.D	●	●	●	◑	●	◑	◑	●	◑	●	●	◑	●
A2.8.E	●	●	●	●	◑	●	●	●	○	●	●	◑	●
A2.8.F	●	●	◑	●	◑	○	●	◑	◑	●	◑	◑	●
A2.8.G	◑	●	◑	●	●	○	○	◑	◑	●	◑	○	◑
A2.8.H	●	●	◑	●	◑	○	◑	◑	◑	●	◑	◑	◑
Overall	●	●	●	●	●	●	●	●	●	◑	◑	◑	●

Table 29. This table shows the results from Integrated Math 1, treated as a series (left chart) and as individual courses (right chart).

PE	Core Plus Math (series)	SIMMS Math (series)	Interactive Math Program (series)	Overall (series)	Core Plus Math (individual)	SIMMS Math (individual)	Interactive Math Program (individual)	Overall (individual)
Solving Problems	●	●	◑	●	●	●	◑	●
M1.1.A	●	●	●	●	●	●	●	●
M1.1.B	●	●	◑	●	●	●	○	●
M1.1.C	●	●	◑	●	●	●	○	●
M1.1.D	●	●	○	●	●	●	○	●
Characteristics and behaviors of functions	●	●	◑	●	○	●	○	◑
M1.2.A	◑	●	○	◑	○	●	○	◑
M1.2.B	●	●	●	●	◑	●	●	●
M1.2.C	●	◑	◑	◑	○	◑	○	◑
M1.2.D	●	◑	○	◑	○	◑	○	○
Linear functions, equations and relationships	●	●	○	◑	●	●	○	◑
M1.3.A	●	●	○	●	●	●	○	●
M1.3.B	◑	◑	◑	◑	○	◑	○	○
M1.3.C	●	◑	○	◑	◑	◑	○	◑
M1.3.D	◑	●	○	◑	◑	●	○	◑
M1.3.E	●	●	◑	●	●	●	○	●
M1.3.F	●	●	●	●	●	●	●	●
M1.3.G	●	○	○	○	◑	○	○	○
M1.3.H	●	◑	○	◑	●	◑	○	◑
Proportionality, similarity, and geometric reasoning	●	○	◑	◑	◑	○	◑	◑
M1.4.A	●	○	○	◑	◑	○	○	○
M1.4.B	●	◑	●	●	●	◑	●	●
M1.4.C	●	○	●	◑	◑	○	●	◑
M1.4.D	●	◑	●	●	◑	◑	●	◑
M1.4.E	●	○	◑	◑	○	○	◑	○
M1.4.F	●	○	●	●	◑	○	●	◑
M1.4.G	●	○	○	○	○	○	○	○
Data and distributions	●	◑	●	●	●	◑	●	●
M1.5.A	●	●	●	●	●	●	●	●
M1.5.B	●	○	●	◑	●	○	●	◑
M1.5.C	●	●	●	●	◑	●	●	●
Numbers, expressions and operations	●	○	◑	◑	◑	○	○	◑
M1.6.A	◑	○	○	○	◑	○	○	○
M1.6.B	◑	○	◑	○	◑	○	○	○
M1.6.C	●	◑	●	●	●	◑	◑	●
M1.6.D	●	◑	●	●	○	◑	◑	◑
Additional Key Content	●	◑	○	◑	●	◑	○	◑
M1.7.A	●	◑	○	◑	●	◑	○	◑
M1.7.B	●	◑	○	◑	◑	◑	○	○
M1.7.C	●	○	○	◑	●	○	○	◑
M1.7.D	◑	●	○	◑	◑	●	○	◑
Reasoning, Problem Solving, and Communication	●	●	●	●	●	●	●	●
M1.8.A	●	●	●	●	●	●	●	●
M1.8.B	●	●	●	●	●	●	●	●
M1.8.C	●	●	●	●	●	●	●	●
M1.8.D	●	●	●	●	●	●	●	●
M1.8.E	●	●	●	●	●	●	●	●
M1.8.F	●	●	●	●	●	●	●	●
M1.8.G	●	●	●	●	●	●	●	●
M1.8.H	●	◑	●	●	●	◑	●	●
Overall	●	◑	◑	●	●	◑	◑	◑

Table 30. This table shows the results from Integrated Math 2, treated as a series (left chart) and as individual courses (right chart).

PE	Core Plus Math	SIMMS Math	Interactive Math Program	Overall
Modeling situations and solving problems	●	●	●	●
M2.1.A	●	●	●	●
M2.1.B	●	●	●	●
M2.1.C	●	◑	◑	◑
M2.1.D	●	●	●	●
M2.1.E	○	●	●	●
Quadratic functions, equations, and relationships	●	○	◑	◑
M2.2.A	●	●	●	●
M2.2.B	●	●	●	●
M2.2.C	●	●	●	●
M2.2.D	●	○	○	○
M2.2.E	●	○	○	○
M2.2.F	●	○	○	○
M2.2.G	●	○	○	○
M2.2.H	○	●	◑	◑
Conjectures and proofs	●	●	○	◑
M2.3.A	●	●	○	●
M2.3.B	●	◑	○	◑
M2.3.C	◑	●	○	◑
M2.3.D	◑	●	○	◑
M2.3.E	●	●	●	●
M2.3.F	●	●	●	●
M2.3.G	●	●	●	●
M2.3.H	●	●	◑	●
M2.3.I	○	◑	○	○
M2.3.J	●	●	○	◑
M2.3.K	●	◑	○	◑
M2.3.L	●	○	○	◑
M2.3.M	◑	○	○	○
Probability	●	●	●	●
M2.4.A	●	●	●	●
M2.4.B	●	●	◑	●
M2.4.C	○	●	●	●
M2.4.D	●	●	◑	●
Additional Key Content	●	◑	◑	◑
M2.5.A	●	○	●	◑
M2.5.B	◑	○	●	◑
M2.5.C	●	◑	◑	◑
M2.5.D	●	●	○	●
Reasoning, Problem Solving, and Communication	●	●	●	●
M2.6.A	●	●	●	●
M2.6.B	●	●	●	●
M2.6.C	●	●	●	●
M2.6.D	●	●	●	●
M2.6.E	●	●	●	●
M2.6.F	●	●	●	●
M2.6.G	●	●	●	●
M2.6.H	●	●	●	●
Overall	●	●	◑	●

PE	Core Plus Math	SIMMS Math	Interactive Math Program	Overall
Modeling situations and solving problems	●	●	◑	●
M2.1.A	●	●	●	●
M2.1.B	●	●	◑	●
M2.1.C	◑	◑	○	◑
M2.1.D	●	●	○	●
M2.1.E	○	●	◑	◑
Quadratic functions, equations, and relationships	◑	○	○	○
M2.2.A	●	○	○	◑
M2.2.B	●	○	○	◑
M2.2.C	○	○	○	○
M2.2.D	●	○	○	○
M2.2.E	◑	○	○	○
M2.2.F	○	○	○	○
M2.2.G	○	○	○	○
M2.2.H	○	●	○	○
Conjectures and proofs	○	◑	○	○
M2.3.A	○	●	○	◑
M2.3.B	○	◑	○	○
M2.3.C	○	◑	○	○
M2.3.D	○	●	○	○
M2.3.E	○	●	○	○
M2.3.F	○	●	○	◑
M2.3.G	◑	●	◑	◑
M2.3.H	●	◑	◑	◑
M2.3.I	○	◑	○	○
M2.3.J	○	●	○	○
M2.3.K	○	◑	○	○
M2.3.L	●	○	○	○
M2.3.M	◑	○	○	○
Probability	◑	●	○	◑
M2.4.A	●	●	○	◑
M2.4.B	●	●	○	◑
M2.4.C	○	●	◑	◑
M2.4.D	○	○	○	○
Additional Key Content	◑	○	○	○
M2.5.A	◑	○	○	○
M2.5.B	◑	○	◑	○
M2.5.C	●	○	○	○
M2.5.D	○	●	○	◑
Reasoning, Problem Solving, and Communication	●	●	●	●
M2.6.A	●	●	●	●
M2.6.B	●	●	●	●
M2.6.C	●	●	●	●
M2.6.D	●	●	●	●
M2.6.E	●	●	●	●
M2.6.F	●	●	●	●
M2.6.G	●	●	●	●
M2.6.H	●	●	●	●
Overall	◑	◑	○	◑

Table 31. This table shows the results from Integrated Math 3, treated as a series (left chart) and as individual courses (right chart).

PE	Core Plus Math	SIMMS Math	Interactive Math Program	Overall		Core Plus Math	SIMMS Math	Interactive Math Program	Overall
Solving Problems	●	●	◑	●		●	◑	◑	●
M3.1.A	●	●	●	●		●	●	●	●
M3.1.B	●	●	●	●		●	○	●	●
M3.1.C	●	◑	◑	◑		●	◑	◑	◑
M3.1.D	●	●	◑	●		●	●	◑	●
M3.1.E	●	◑	○	◑		●	◑	○	◑
Transformations and functions	●	●	○	◑		○	●	○	○
M3.2.A	●	●	○	◑		◑	●	○	○
M3.2.B	●	●	○	●		●	●	○	◑
M3.2.C	●	●	○	◑		○	●	○	○
M3.2.D	●	◑	○	◑		○	◑	○	○
M3.2.E	○	●	○	◑		○	●	○	◑
Functions and modeling	◑	◑	○	◑		◑	◑	○	○
M3.3.A	●	◑	◑	●		●	◑	○	◑
M3.3.B	○	○	◑	○		○	○	○	○
M3.3.C	●	◑	◑	●		●	◑	○	◑
M3.3.D	○	○	○	○		○	○	○	○
M3.3.E	●	●	○	◑		●	●	○	◑
M3.3.F	●	◑	○	◑		●	○	○	○
M3.3.G	○	●	●	◑		○	○	●	○
Quantifying variability	◑	●	○	◑		◑	●	○	○
M3.4.A	●	●	●	●		●	●	○	●
M3.4.B	○	●	○	○		○	◑	○	○
Three-dimensional geometry	○	●	○	◑		○	◑	○	○
M3.5.A	○	○	●	○		○	○	●	○
M3.5.B	●	●	○	◑		○	●	○	◑
M3.5.C	○	○	○	○		○	○	○	○
M3.5.D	●	●	●	●		○	○	○	○
M3.5.E	○	●	●	◑		○	○	○	○
M3.5.F	○	●	○	○		○	●	○	○
Algebraic properties	●	○	○	○		◑	○	○	○
M3.6.A	◑	○	○	○		◑	○	○	○
M3.6.B	●	●	○	◑		○	○	○	○
M3.6.C	●	○	○	○		●	○	○	○
M3.6.D	●	◑	○	◑		●	◑	○	◑
Additional Key Content	◑	◑	○	◑		◑	○	○	○
M3.7.A	●	●	◑	●		●	○	◑	◑
M3.7.B	●	○	◑	○		○	○	◑	○
M3.7.C	●	●	○	◑		●	○	○	○
M3.7.D	○	●	○	○		○	○	○	○
Reasoning, Problem Solving, and Communication	●	●	●	●		●	●	●	●
M3.8.A	●	●	●	●		●	●	●	●
M3.8.B	●	●	●	●		●	●	●	●
M3.8.C	●	●	●	●		●	●	●	●
M3.8.D	●	●	●	●		●	●	●	●
M3.8.E	●	●	●	●		●	●	●	●
M3.8.F	●	●	●	●		●	●	●	●
M3.8.G	●	●	●	●		●	●	●	●
M3.8.H	●	●	●	●		●	●	●	●
Overall	●	●	○	◑		◑	◑	○	◑

3.3 Program Organization and Design

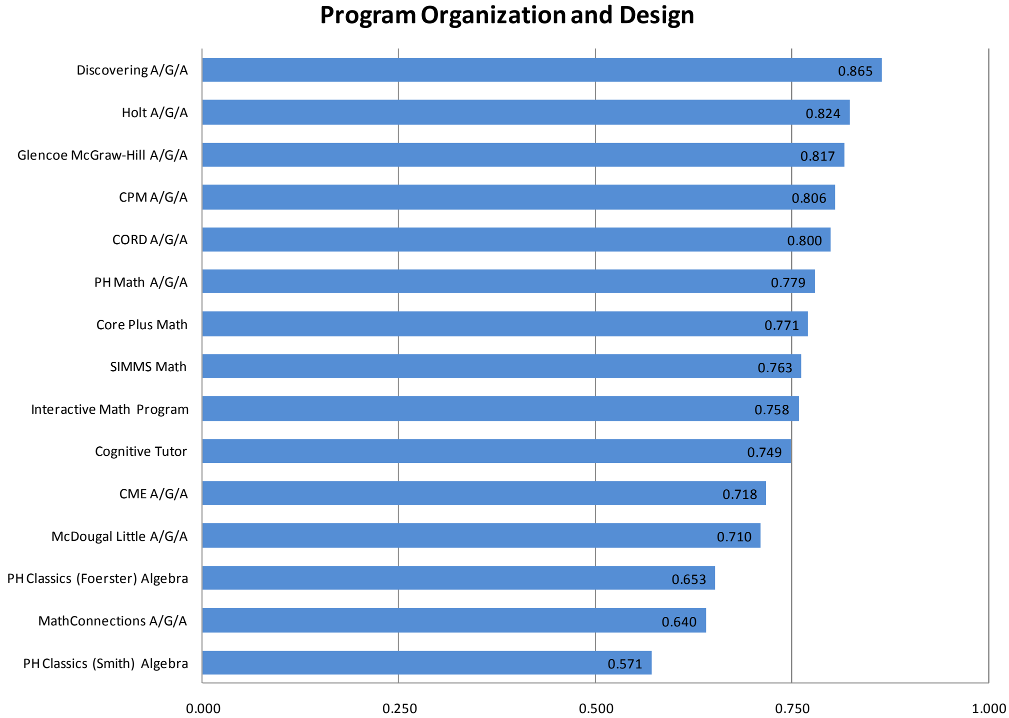


Figure 13. Publisher bundle rank order.

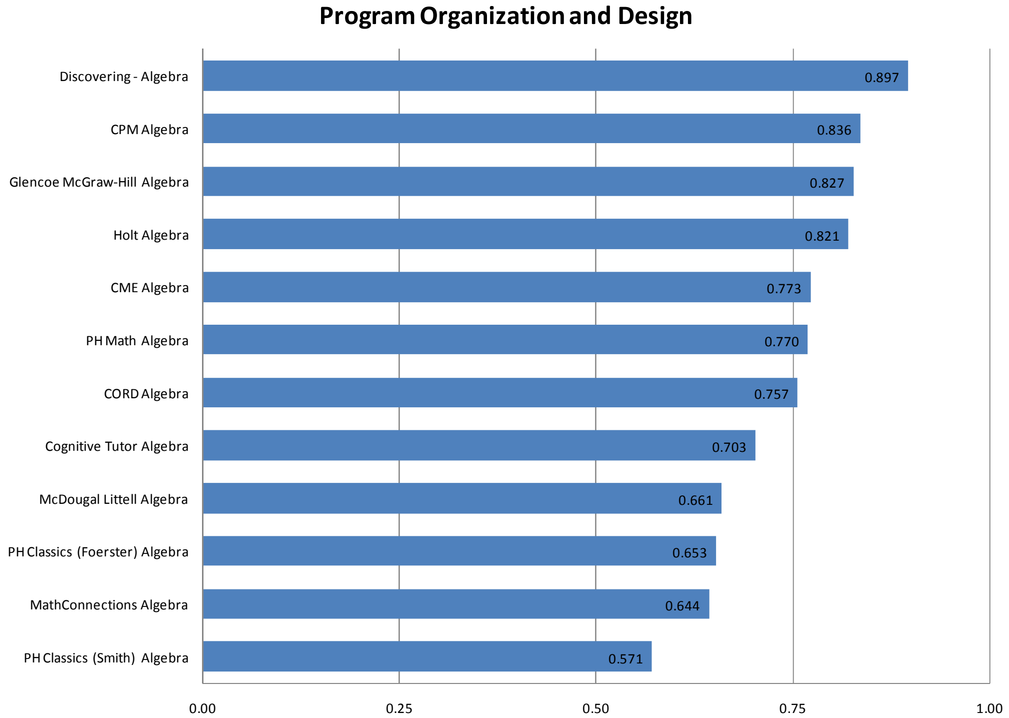


Figure 14. Algebra 1 and 2 series Program Organization and Design scale, in rank order.

Figure 15. Geometry -- Program Organization and Design scale, in rank order.

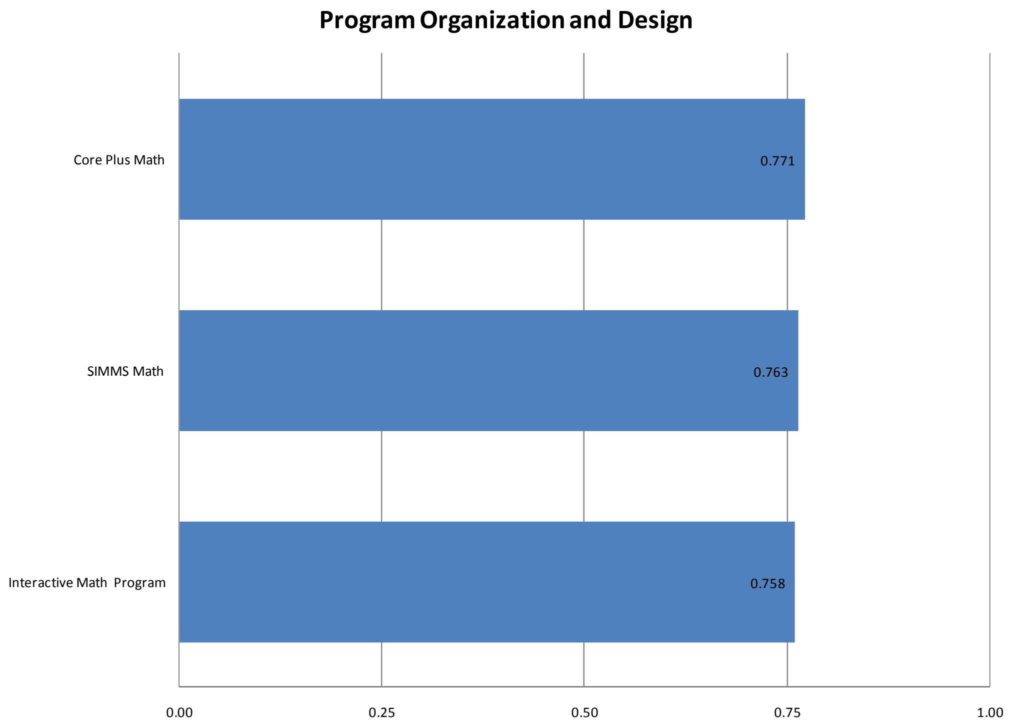


Figure 16. Integrated series Program Organization and Design scale, in rank order.

3.4 Balance of Student Experience

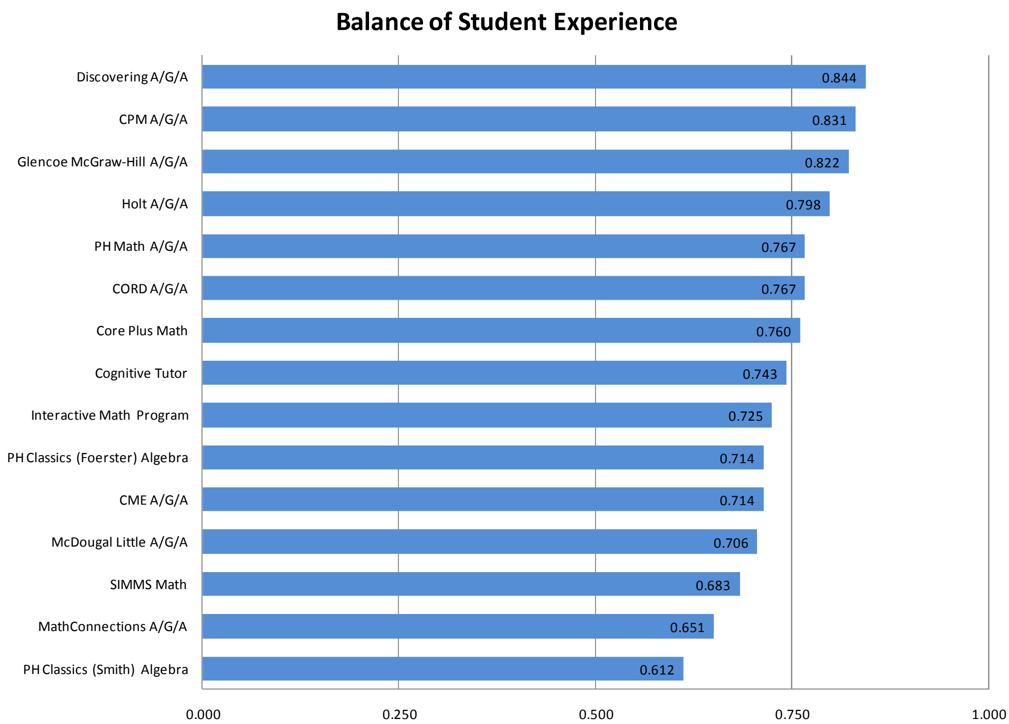


Figure 17. Publisher bundle rank order.

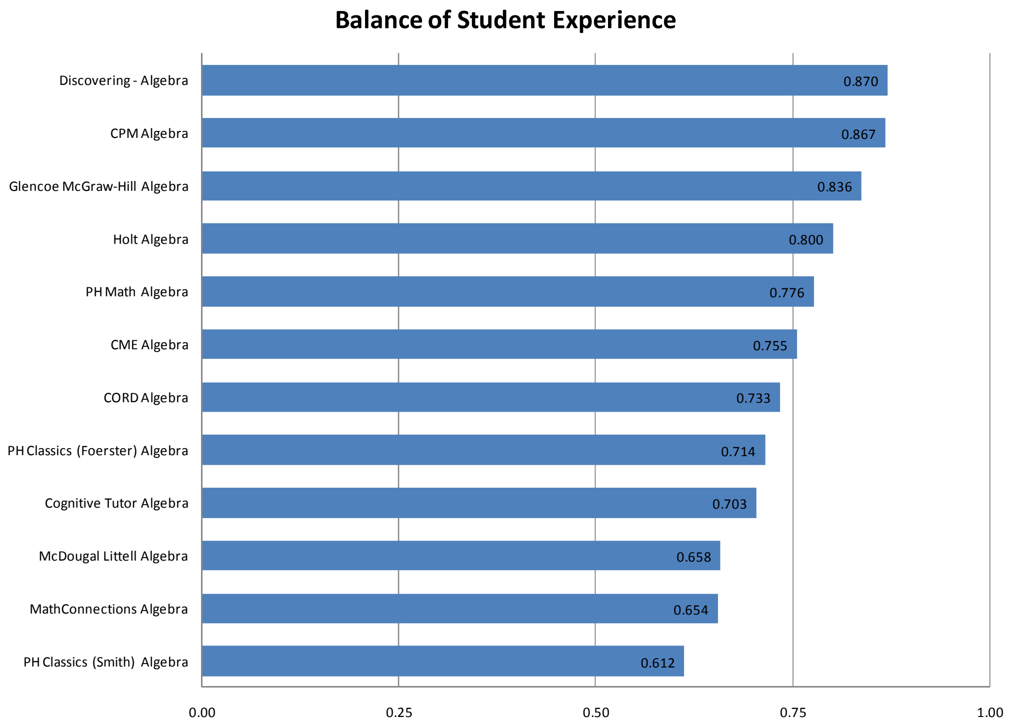


Figure 18. Balance of Student Experience scale for Algebra 1 and 2 series, in rank order.

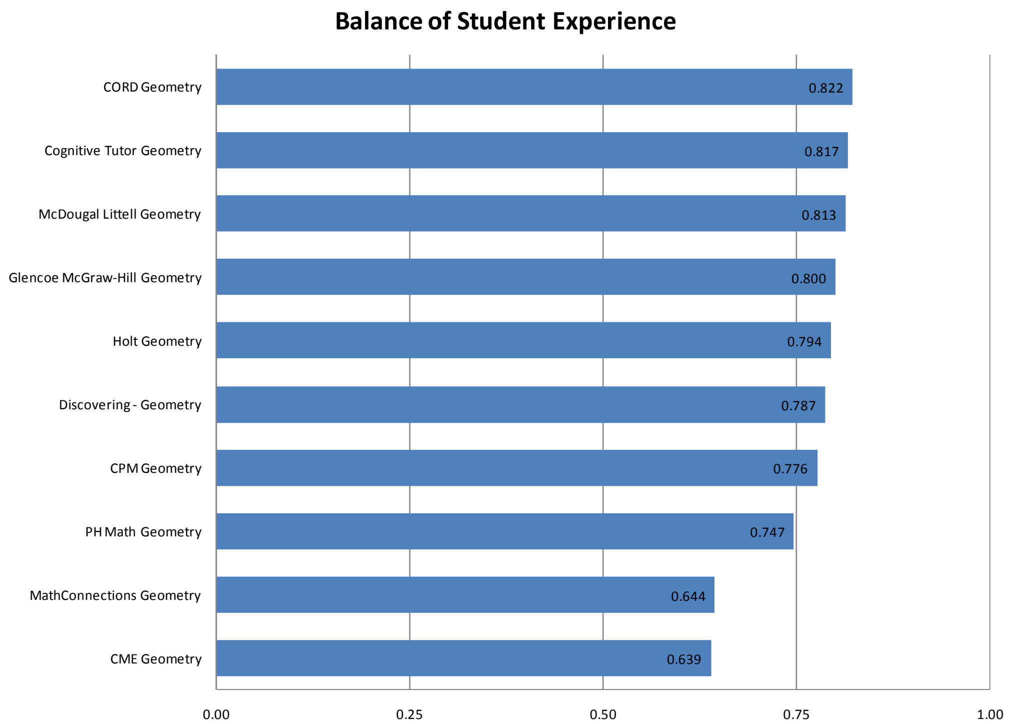


Figure 19. Balance of Student Experience scale for Geometry programs, in rank order.

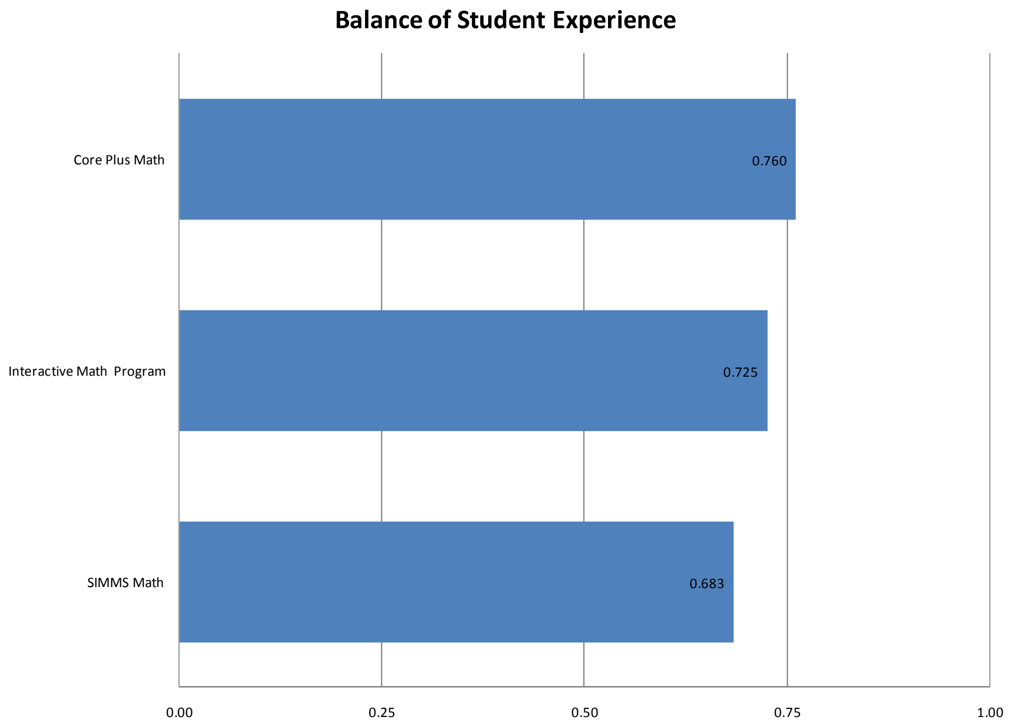


Figure 20. Balance of Student Experience scale for Integrated programs, in rank order.

3.5 Assessment

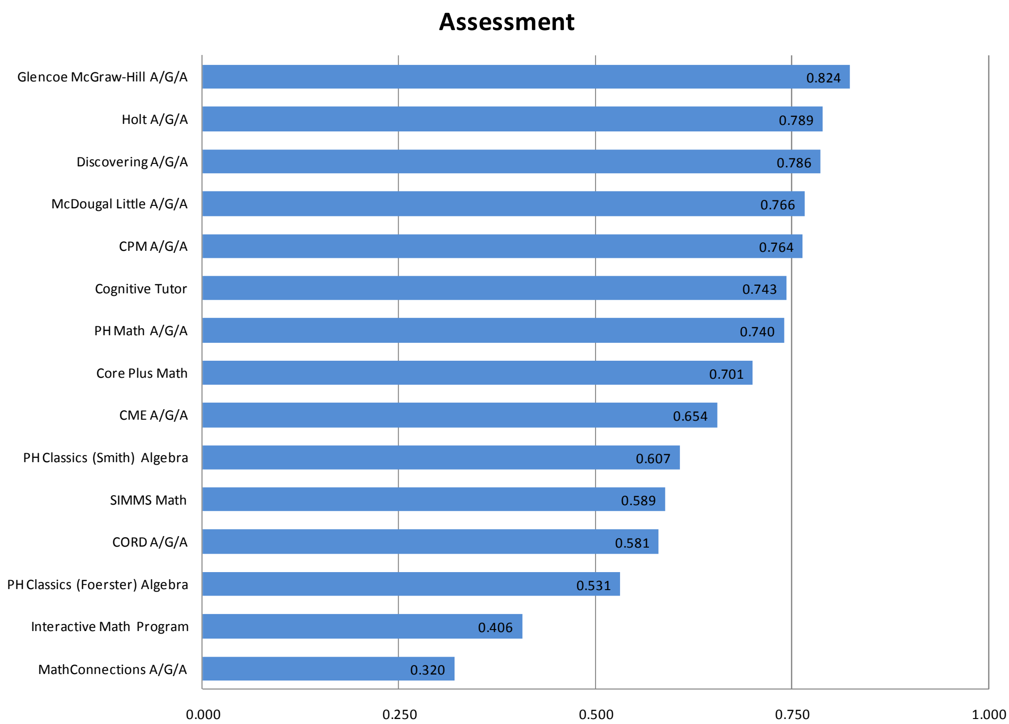


Figure 21. Publisher bundle rank order.

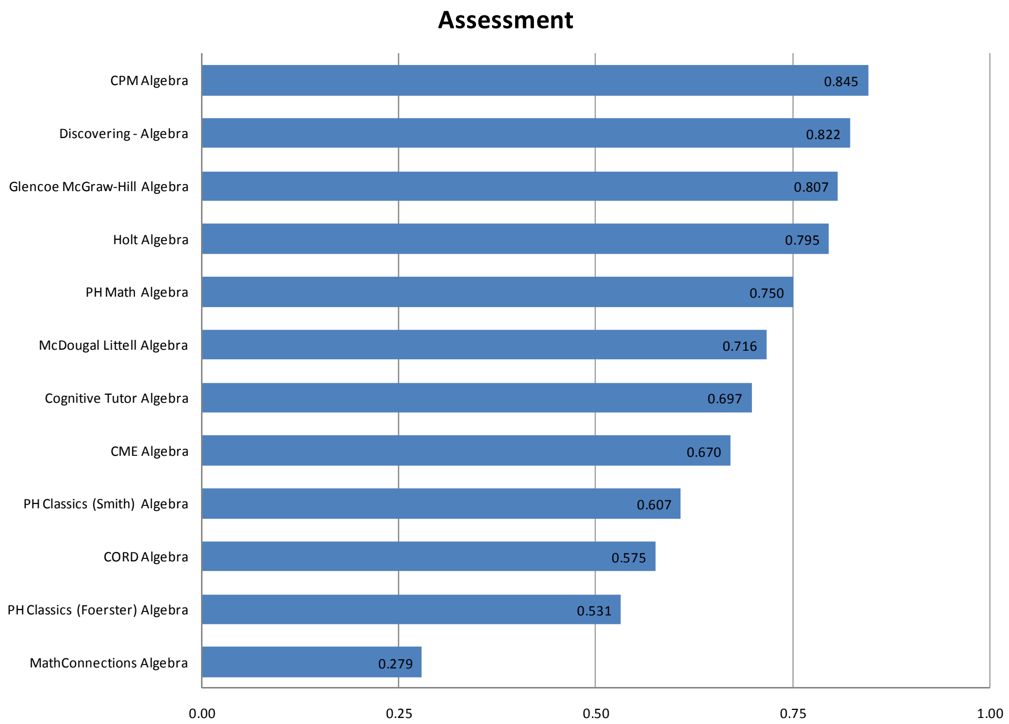


Figure 22. Assessment scale for Algebra 1 and 2 series, in rank order.

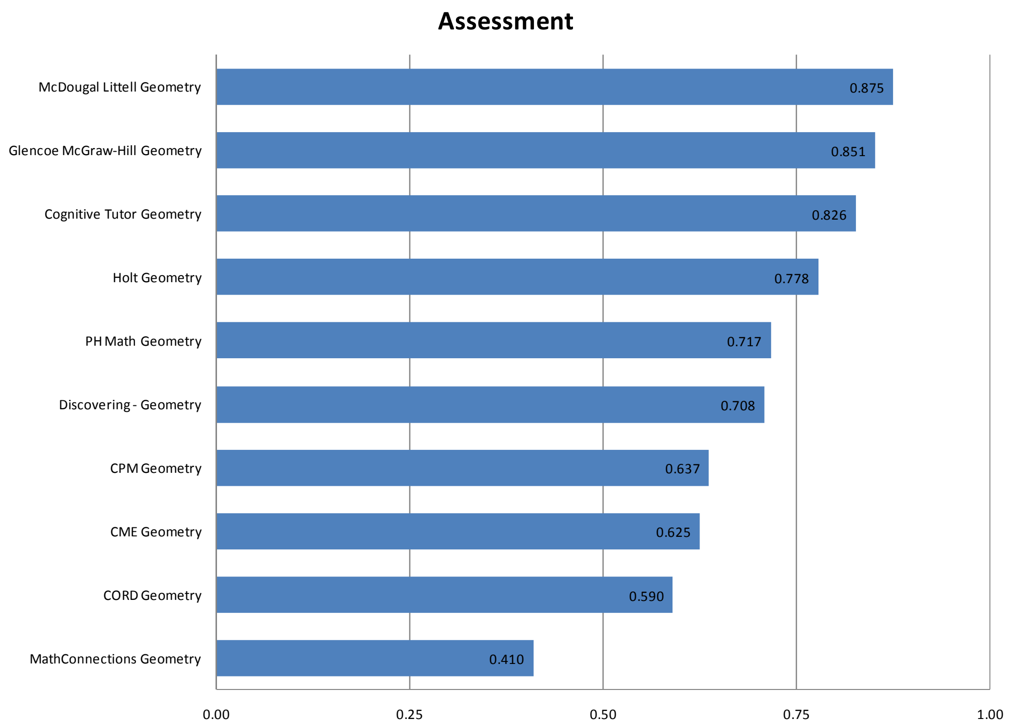


Figure 23. Assessment scale for Geometry programs, in rank order.

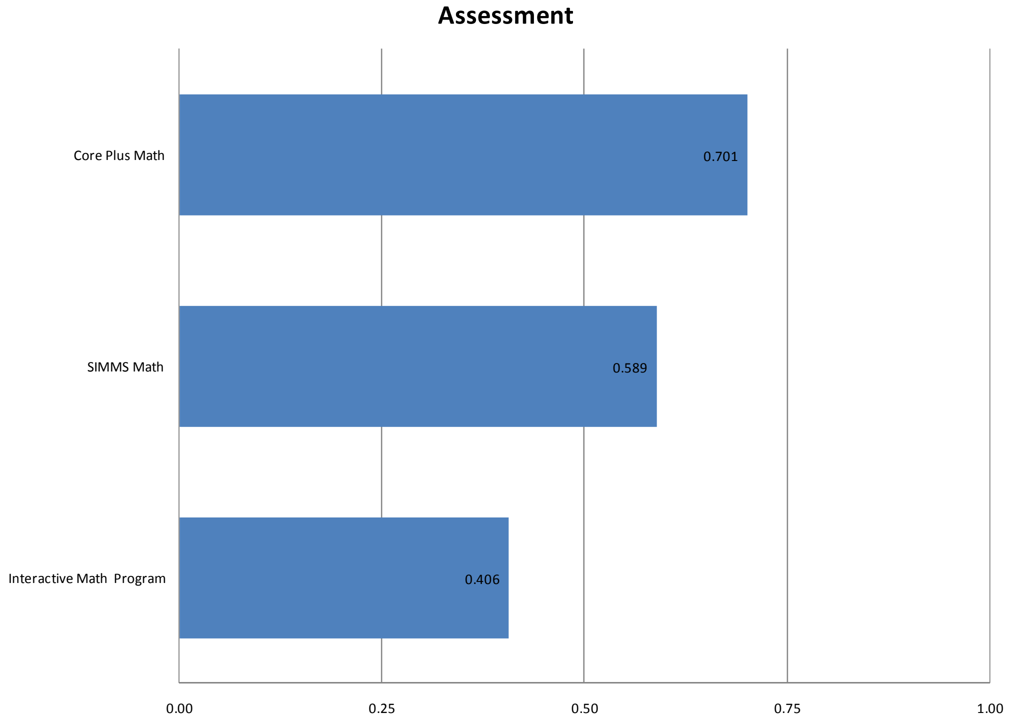


Figure 24. Assessment scale for Integrated programs, in rank order.

3.6 Instructional Planning and Professional Support

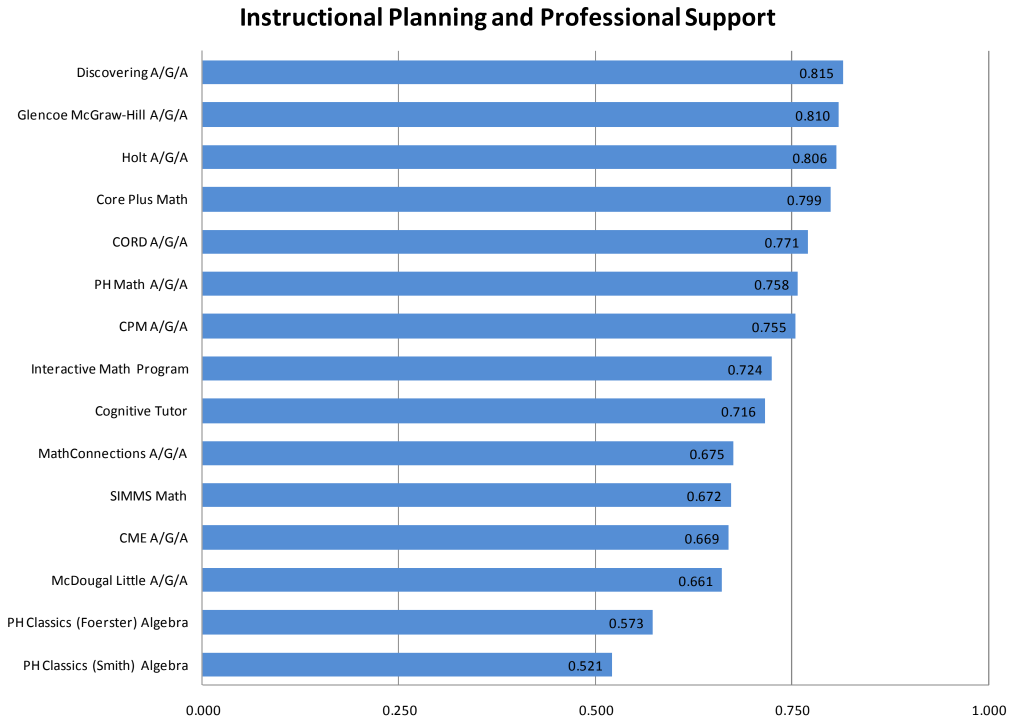


Figure 25. Publisher bundle rank order.

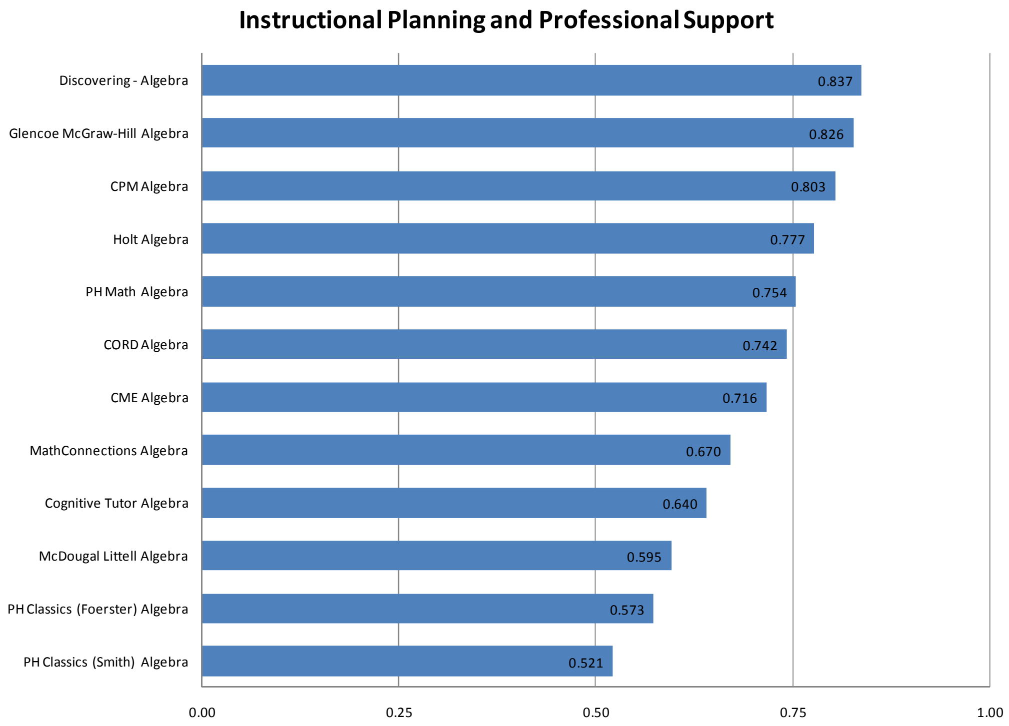


Figure 26. Instructional Planning and Professional Support scale for Algebra 1 and 2 series, in rank order.

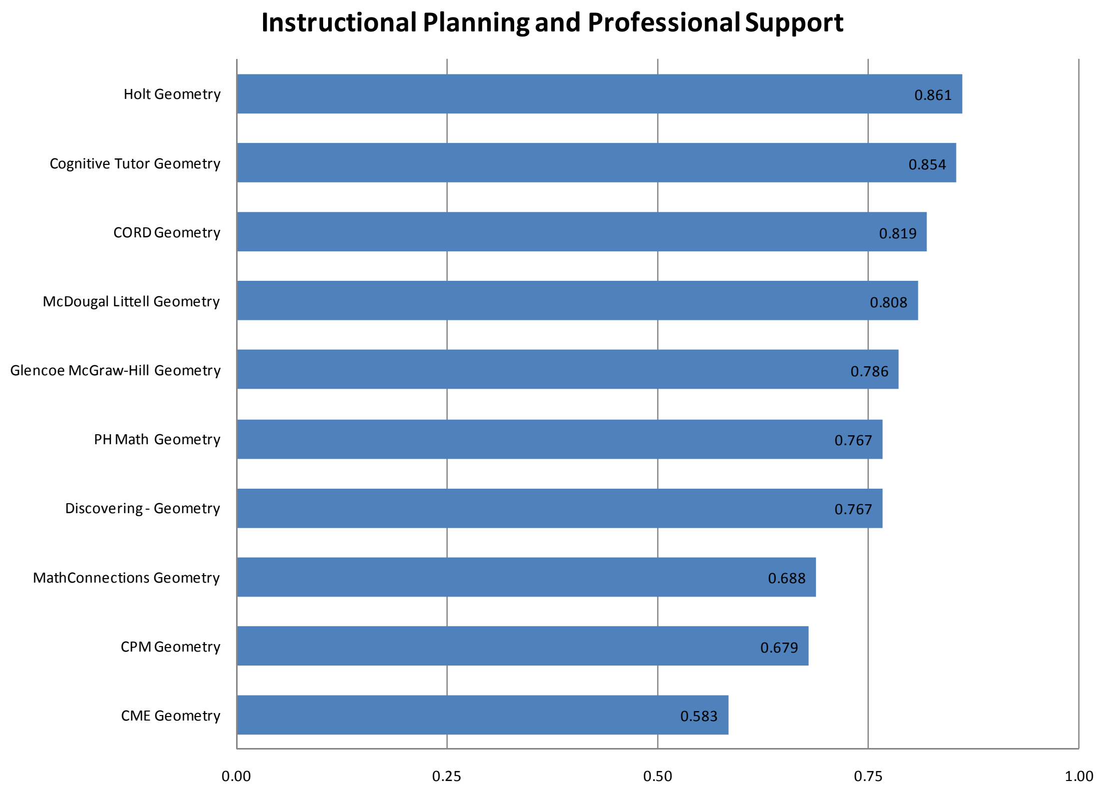


Figure 27. Instructional Planning and Professional Support scale for Geometry programs, in rank order.

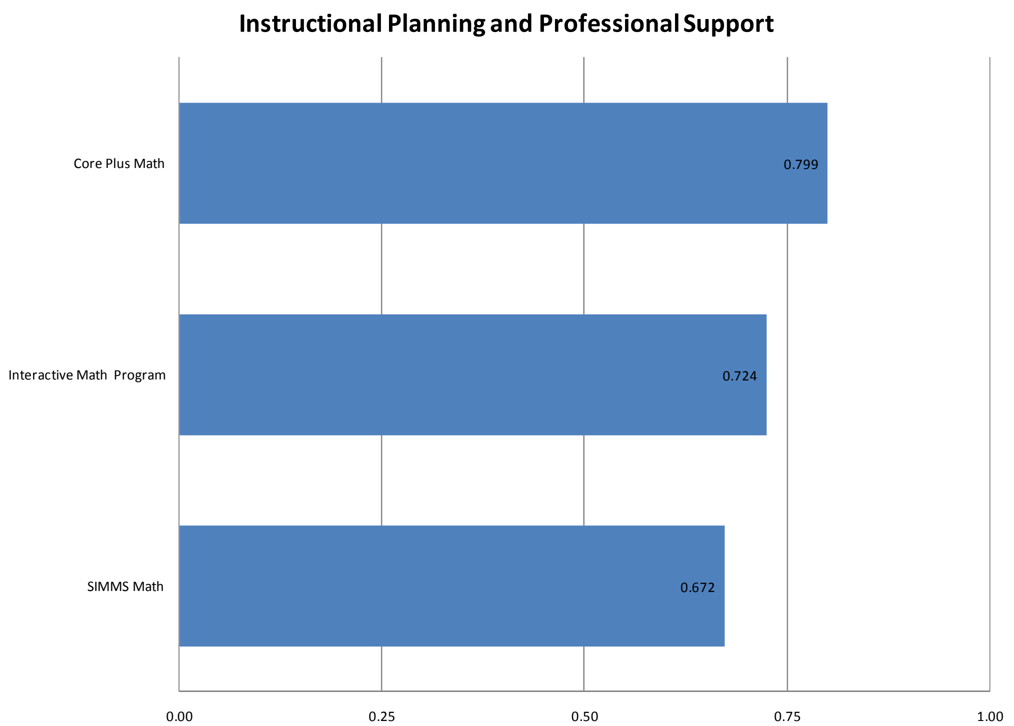


Figure 28. Instructional Planning and Professional Support scale for Integrated programs, in rank order.

3.7 Equity and Access

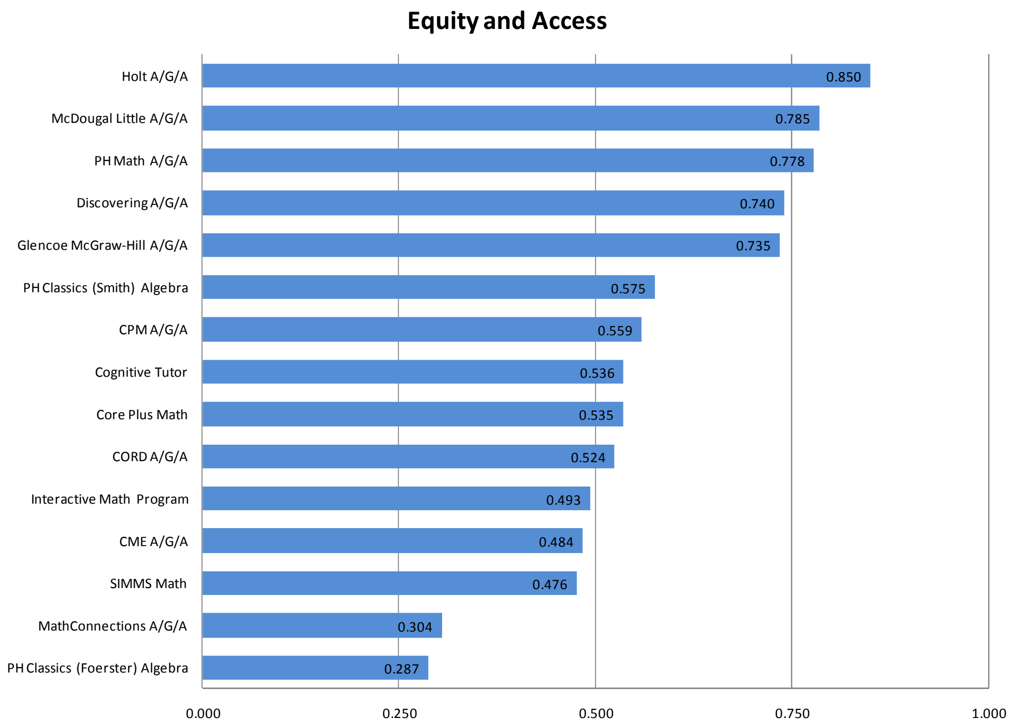


Figure 29. Publisher bundle rank order.

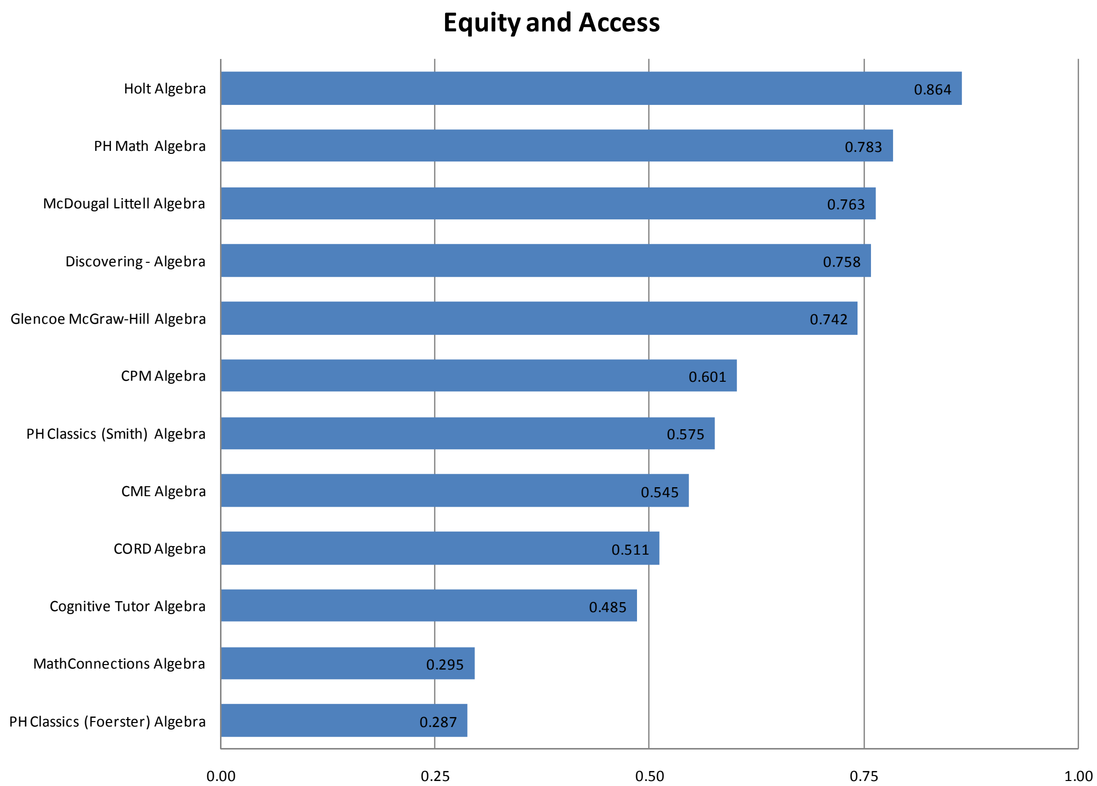


Figure 30. Equity and Access scale results for Algebra 1 and 2 series, in rank order.

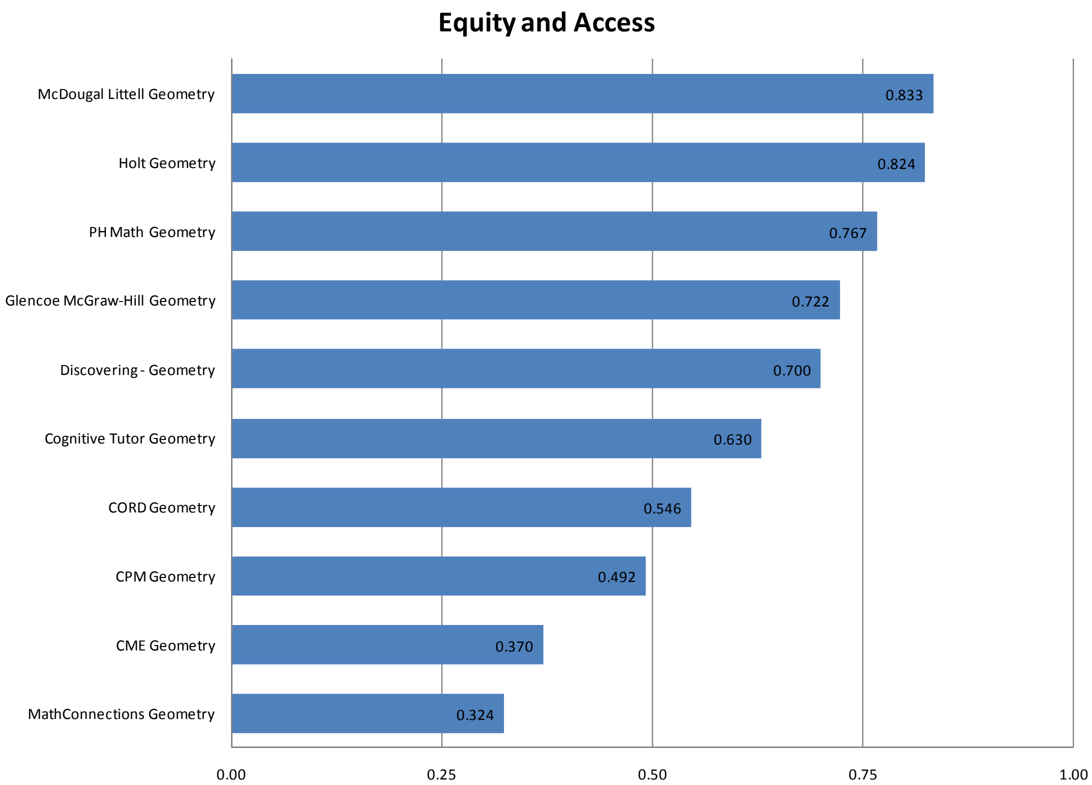


Figure 31. Equity and Access scale results for Geometry programs, in rank order.

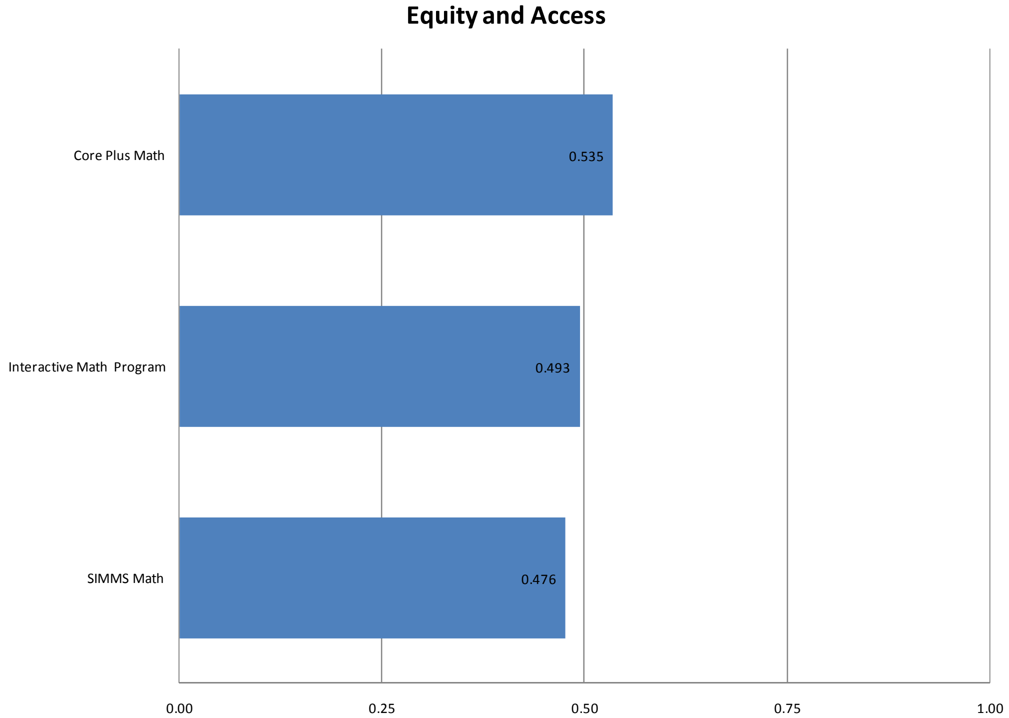
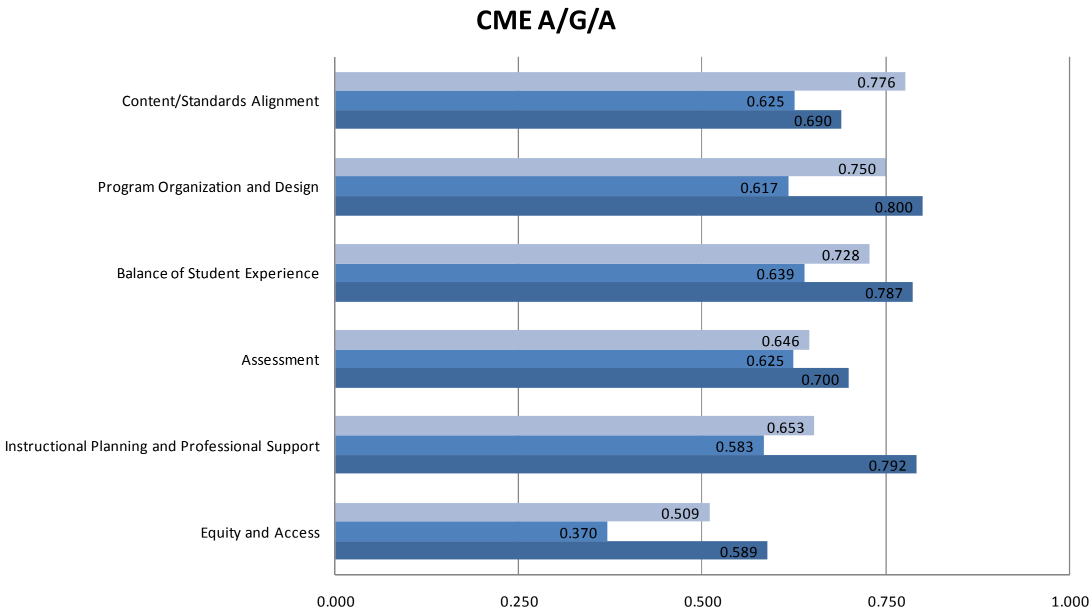


Figure 32. Equity and Access scale results for Integrated programs, in rank order.

3.8 Results of Individual Publisher Series

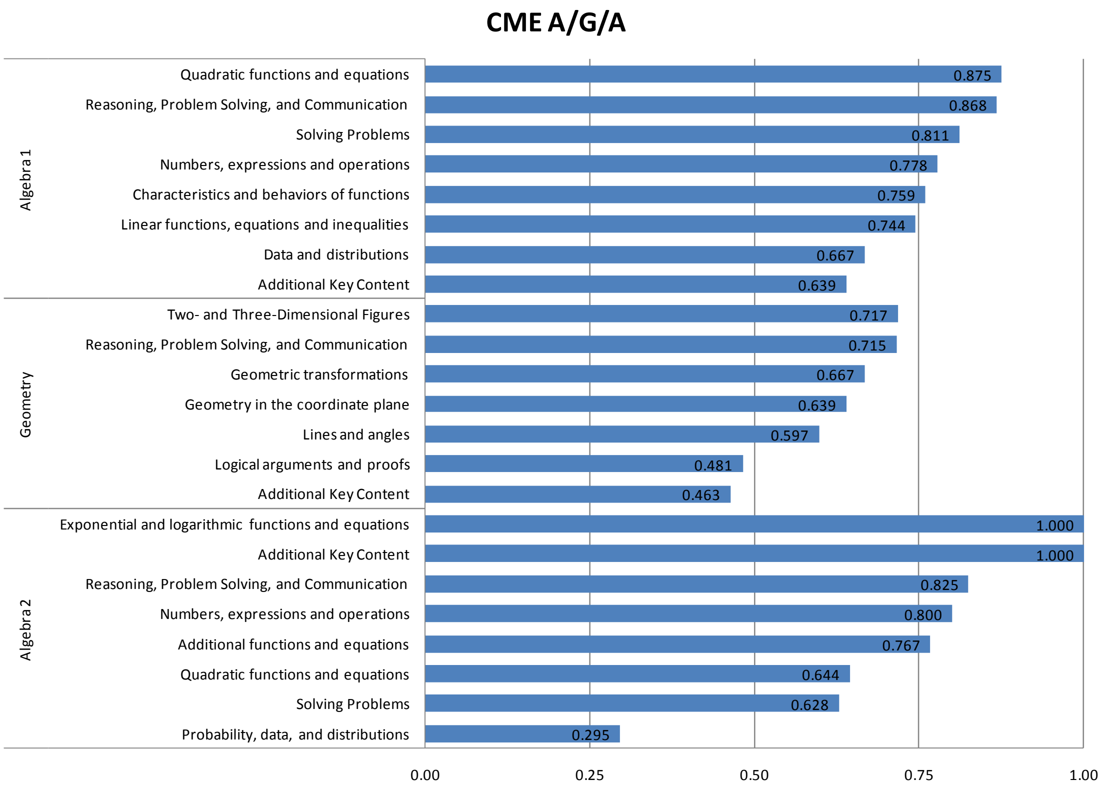
This section presents individual graphs and narrative that describe how the particular publisher series did in the review process. It includes scaled values for each scale, for all courses submitted for review. Note that this section includes results from all programs presented alphabetically.

3.8.1 CME (A/G/A)



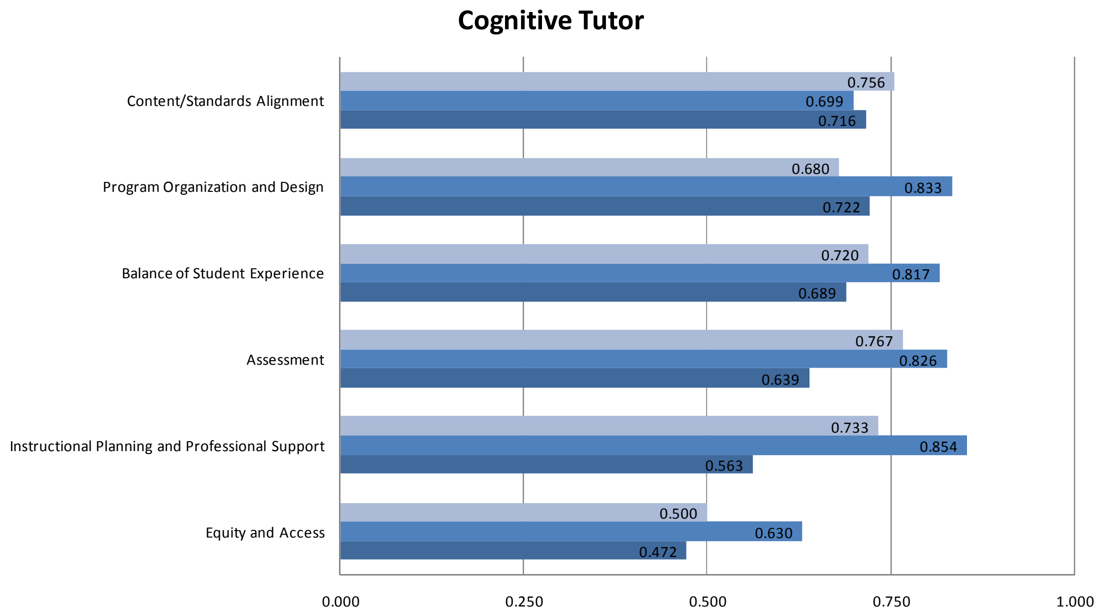
	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.509	0.653	0.646	0.728	0.750	0.776
Geometry	0.370	0.583	0.625	0.639	0.617	0.625
Algebra 2	0.589	0.792	0.700	0.787	0.800	0.690

This graph and chart combination shows each of the scales on the vertical axis, and displays the scaled average score for each course on the horizontal axis. The intent is to see a complete picture of how the program performed at all course levels and all scales.

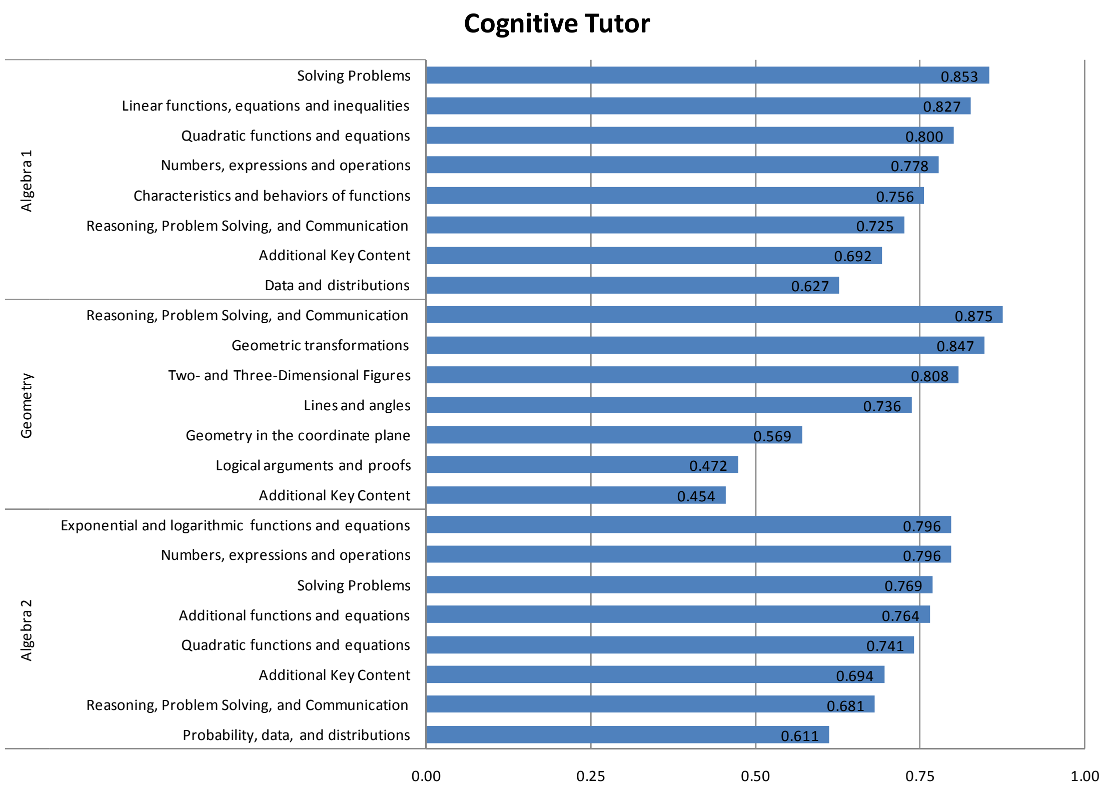


This graph shows the Core Content Areas of the 2008 Revised Washington Standards, organized by course for the program CME. Within each course, the core content areas are organized by average score, from highest to lowest. This graph gives school districts valuable information on broad categories of areas where the series does well, or where it might need to be supplemented.

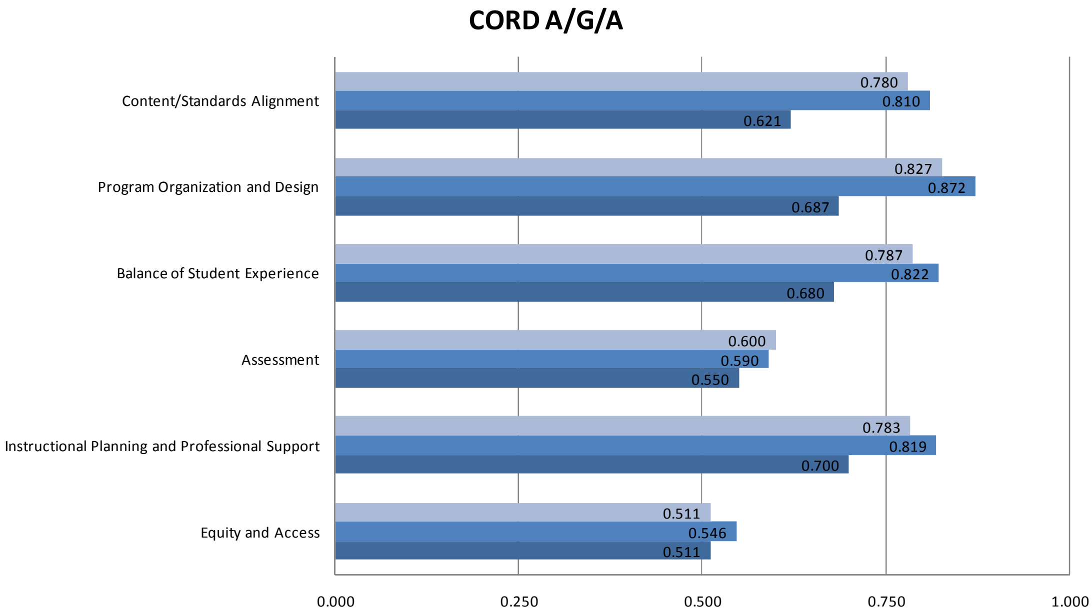
3.8.2 Cognitive Tutor (A/G/A)



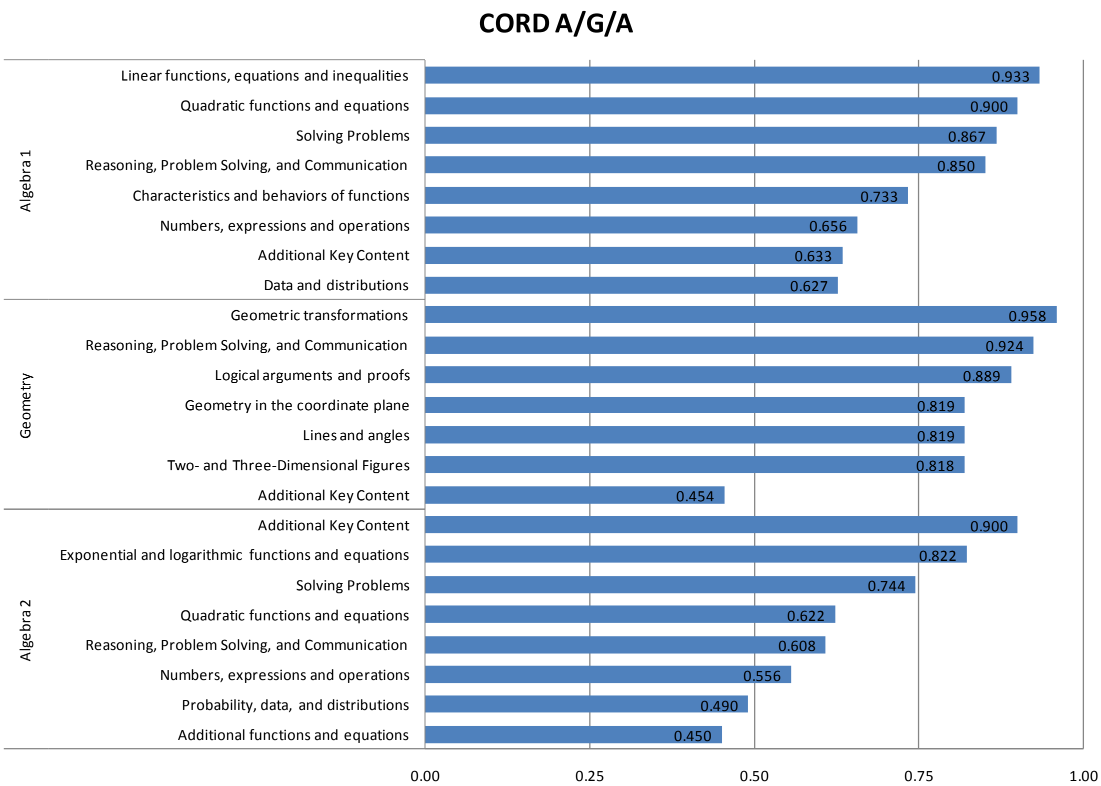
	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.500	0.733	0.767	0.720	0.680	0.756
Geometry	0.630	0.854	0.826	0.817	0.833	0.699
Algebra 2	0.472	0.563	0.639	0.689	0.722	0.716



3.8.3 CORD (A/G/A)

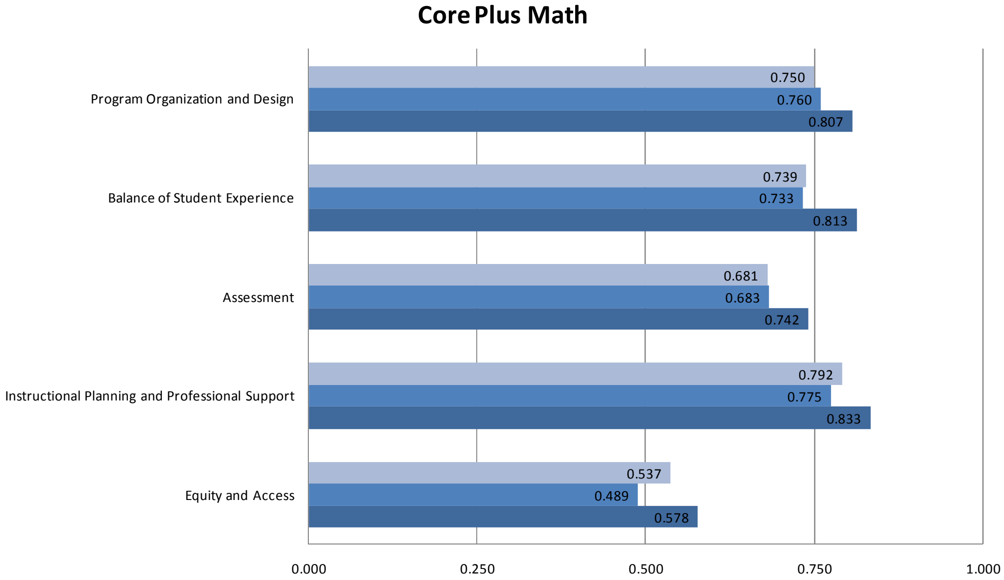


	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.511	0.783	0.600	0.787	0.827	0.780
Geometry	0.546	0.819	0.590	0.822	0.872	0.810
Algebra 2	0.511	0.700	0.550	0.680	0.687	0.621



3.8.4 Core Plus Math (Integrated)

Graphs for the integrated programs are presented differently. Figure 33 shows the results for the Core Plus Math series by scale. Content/Standards Alignment scale results are presented in Figure 34, and reflect treating the product as individual courses and as a series as a whole. This is shown both ways because almost 30% of the time, the integrated texts reviewed met the standards in a course above or below the expected level. One explanation for the high percentage of grade dips may be the placement of the integrated standards in Math 1, 2 and 3. Figure 35 and Figure 36 show core content area results, for individual courses and the series as a whole. All three integrated programs reviewed show results in both formats.



	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design
Math 1	0.537	0.792	0.681	0.739	0.750
Math 2	0.489	0.775	0.683	0.733	0.760
Math 3	0.578	0.833	0.742	0.813	0.807

Figure 33. This graph shows all scales except for content/standards alignment for Math 1, 2 and 3 for Core Plus Math.

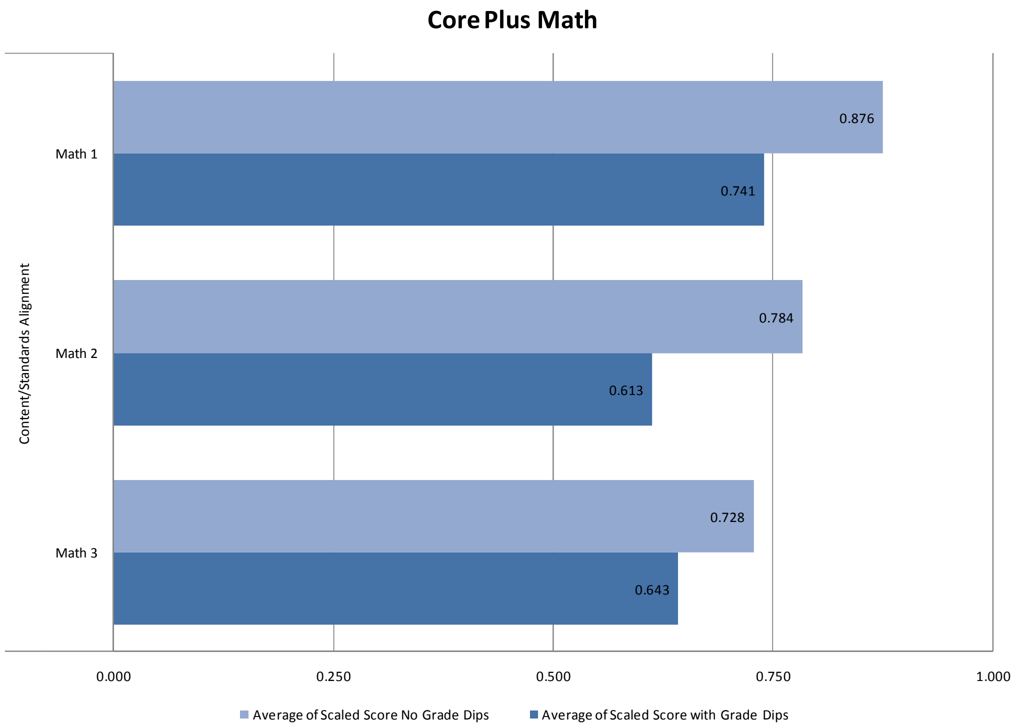


Figure 34. Content/Standards Alignment scale results for the series as a whole (light blue) and for individual courses (dark blue) for Core Plus Math Integrated series.

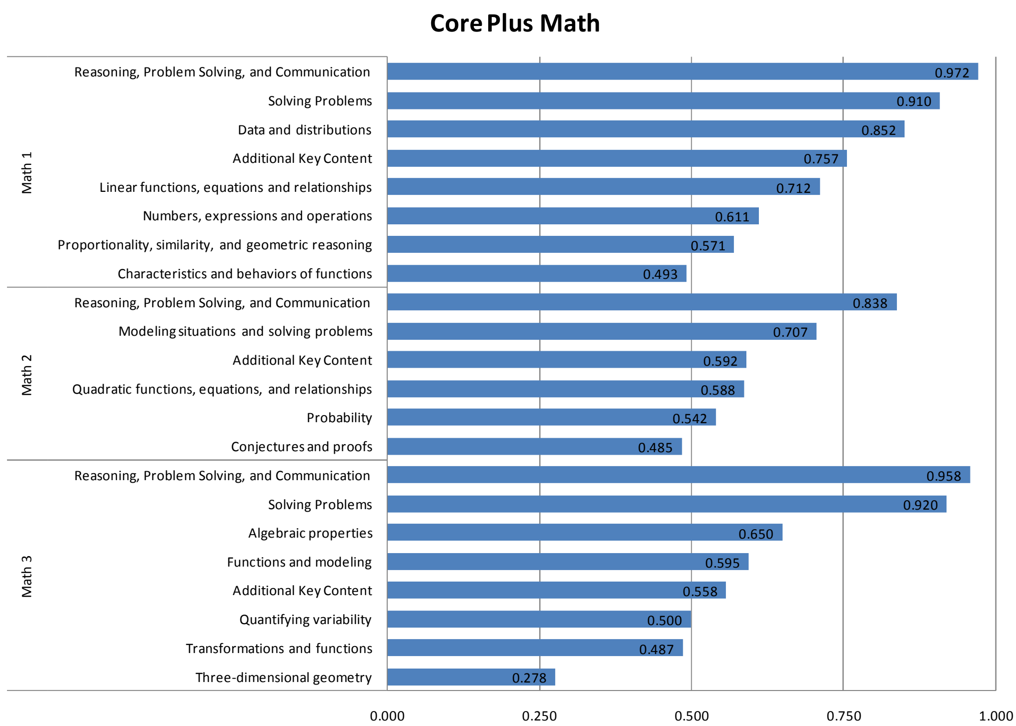


Figure 35. Core Content Area results for individual courses.

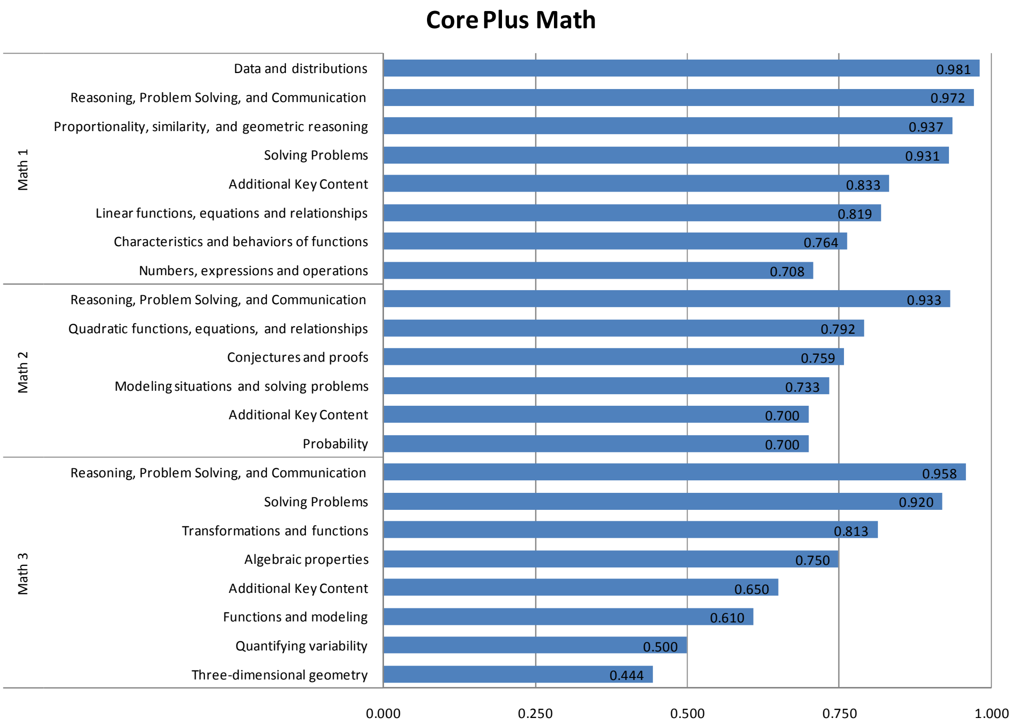
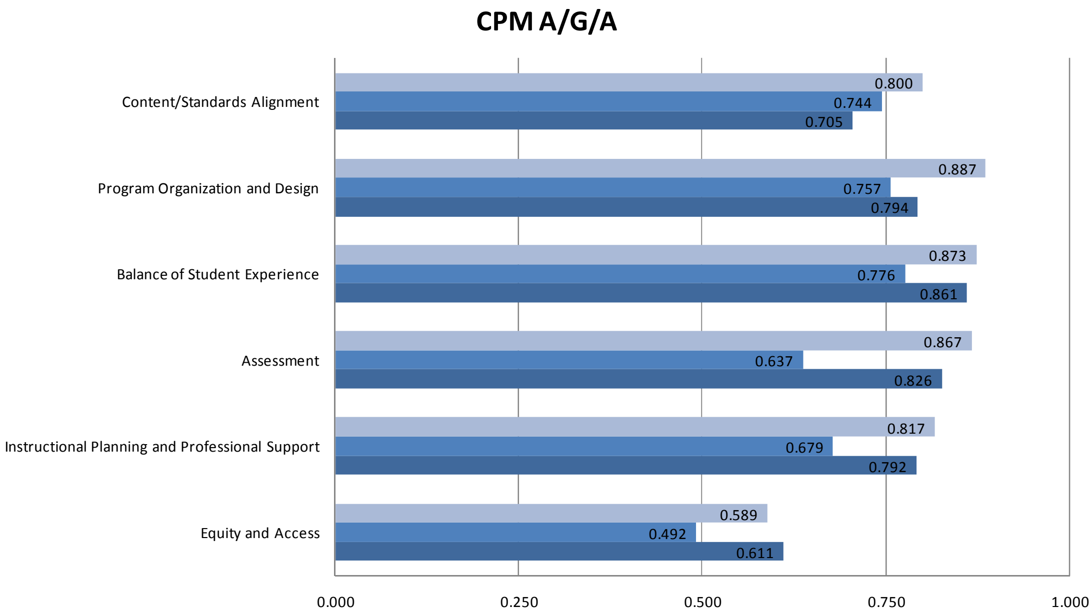
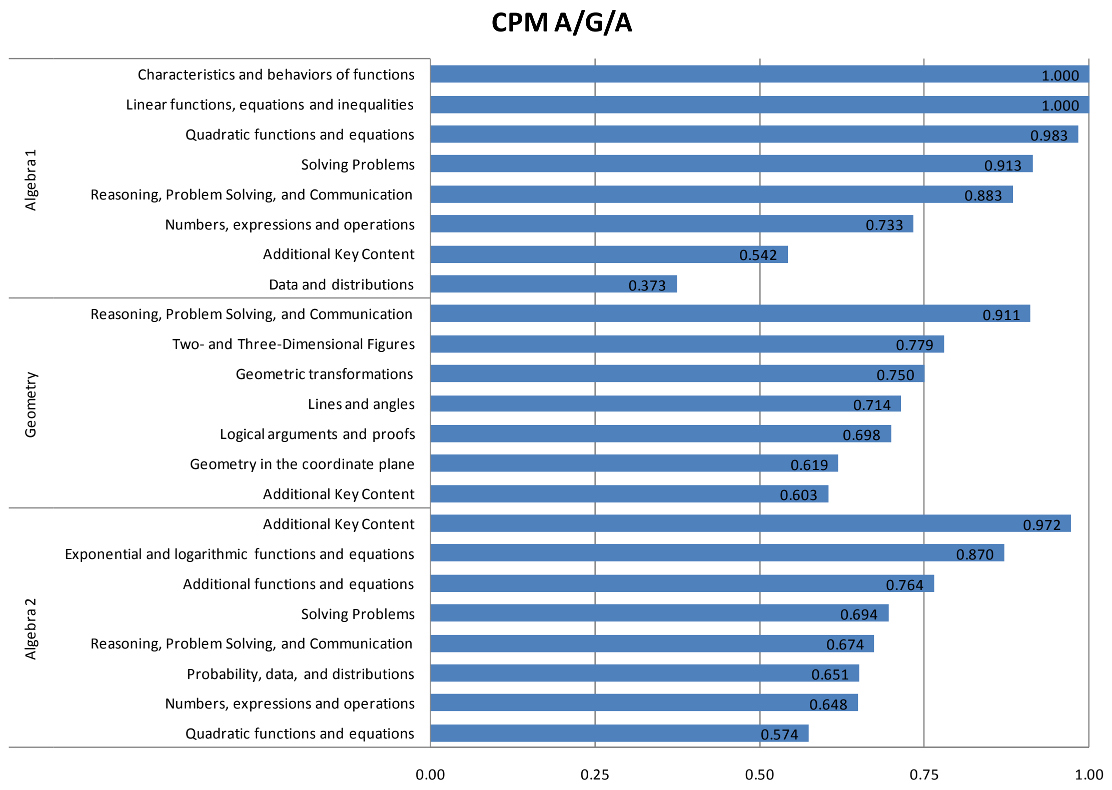


Figure 36. Core Content Area results for the series as a whole.

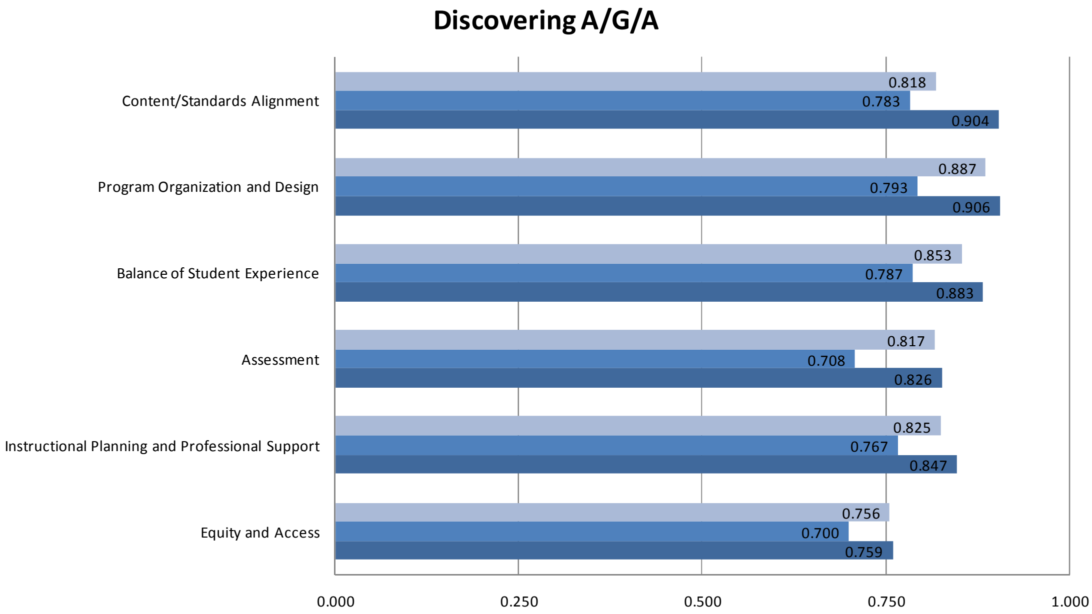
3.8.5 CPM (A/G/A)



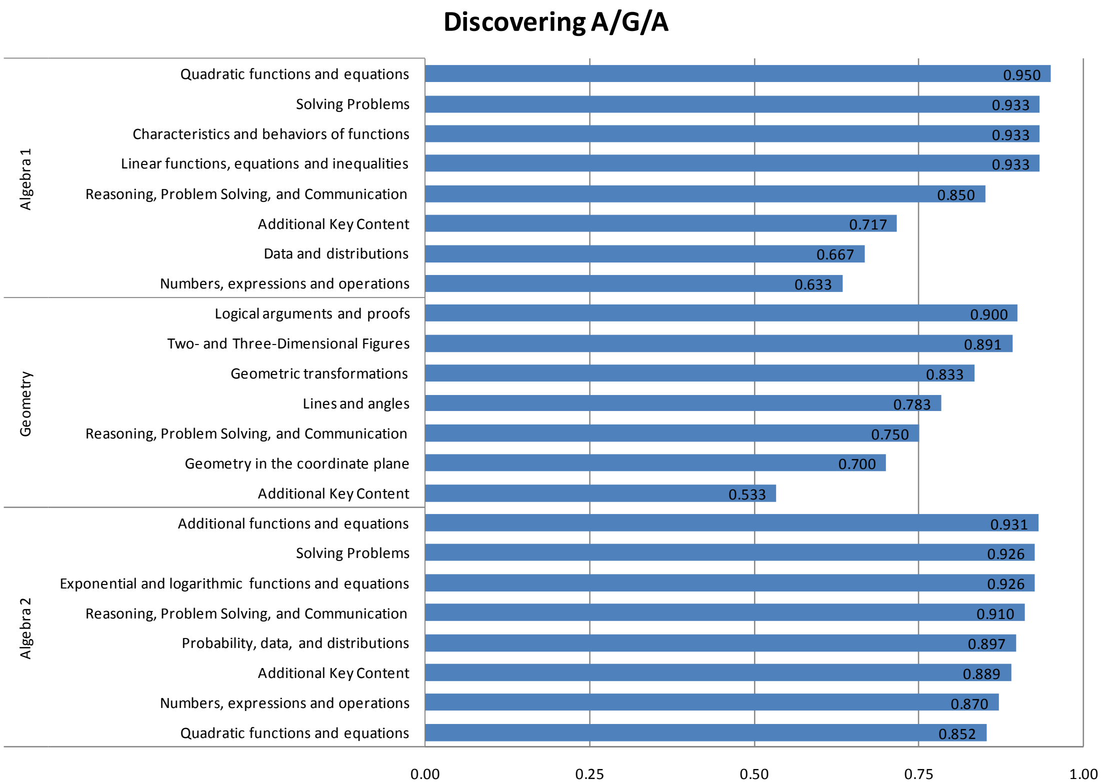
	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.589	0.817	0.867	0.873	0.887	0.800
Geometry	0.492	0.679	0.637	0.776	0.757	0.744
Algebra 2	0.611	0.792	0.826	0.861	0.794	0.705


CPM A/G/A
Algebra 1
Characteristics and behaviors of functions
1.000
Linear functions, equations and inequalities
1.000
Quadratic functions and equations
0.983
Solving Problems
0.913
Reasoning, Problem Solving, and Communication
0.883
Numbers, expressions and operations
0.733
Additional Key Content
0.542
Data and distributions
0.373
Geometry
Reasoning, Problem Solving, and Communication
0.911
Two- and Three-Dimensional Figures
0.779
Geometric transformations
0.750
Lines and angles
0.714
Logical arguments and proofs
0.698
Geometry in the coordinate plane
0.619
Additional Key Content
0.603
Algebra 2
Additional Key Content
0.972
Exponential and logarithmic functions and equations
0.870
Additional functions and equations
0.764
Solving Problems
0.694
Reasoning, Problem Solving, and Communication
0.674
Probability, data, and distributions
0.651
Numbers, expressions and operations
0.648
Quadratic functions and equations
0.574
0.00
0.25
0.50
0.75
1.00

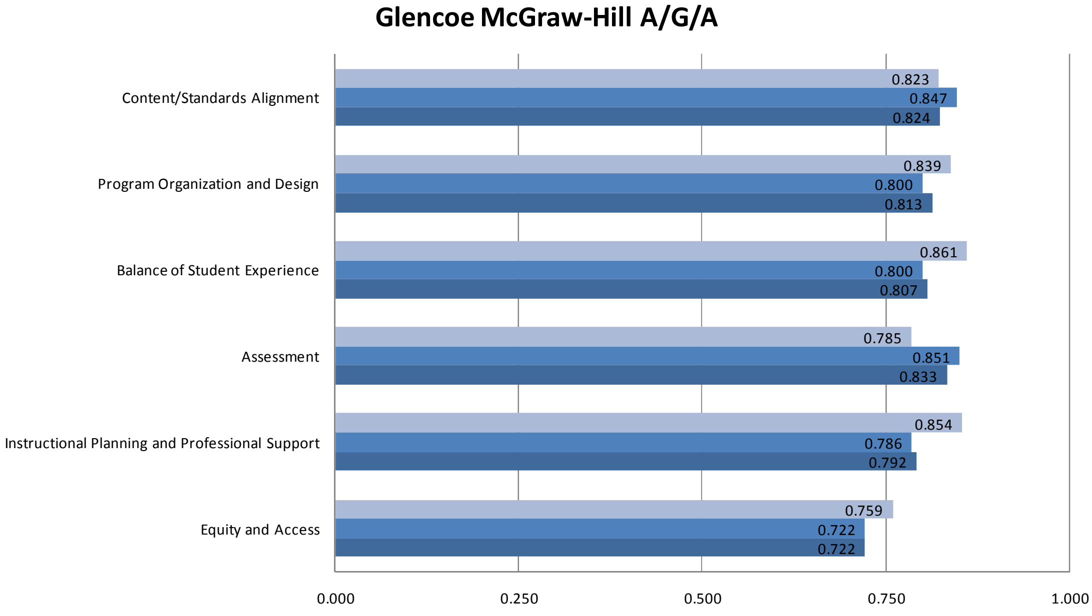
3.8.6 Discovering (A/G/A)



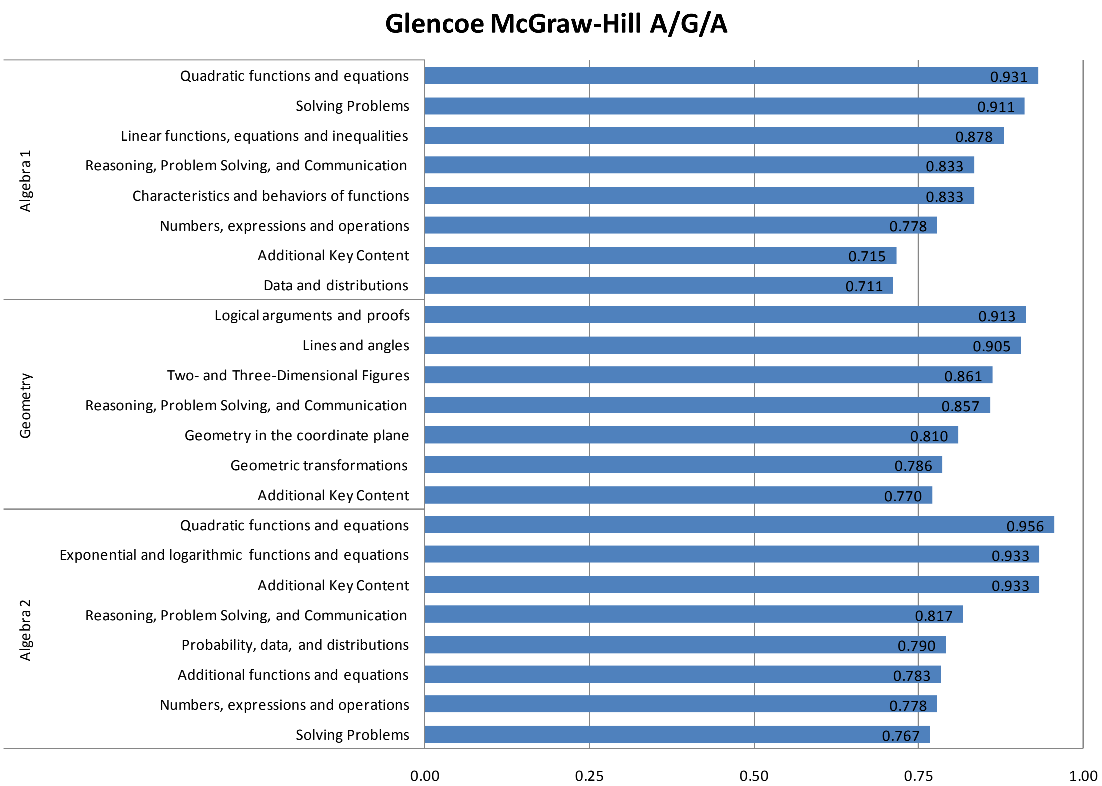
	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.756	0.825	0.817	0.853	0.887	0.818
Geometry	0.700	0.767	0.708	0.787	0.793	0.783
Algebra 2	0.759	0.847	0.826	0.883	0.906	0.904



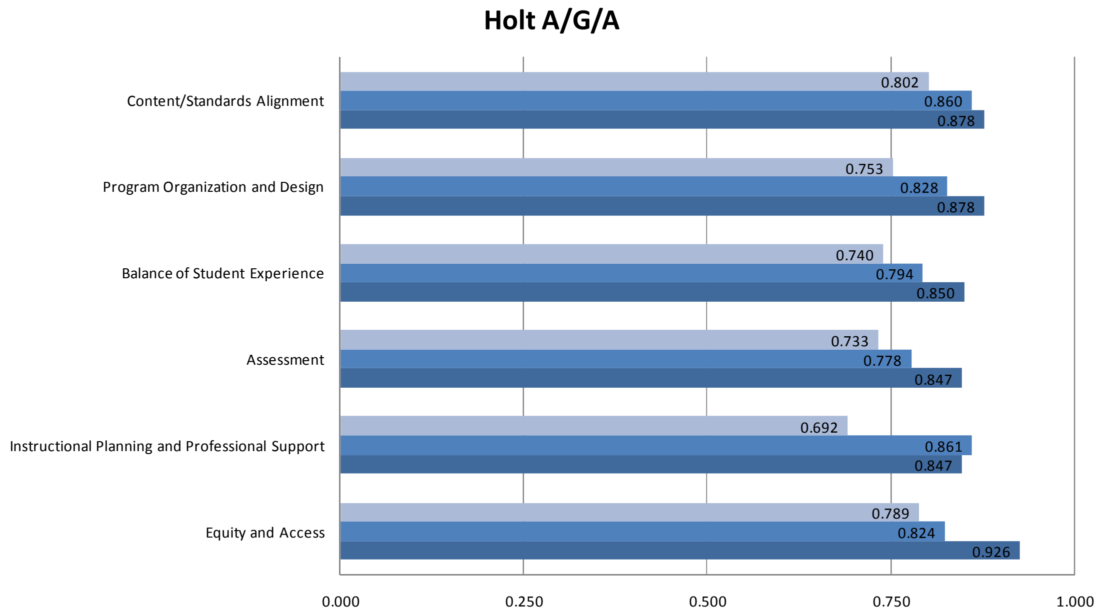
3.8.7 Glencoe McGraw-Hill (A/G/A)



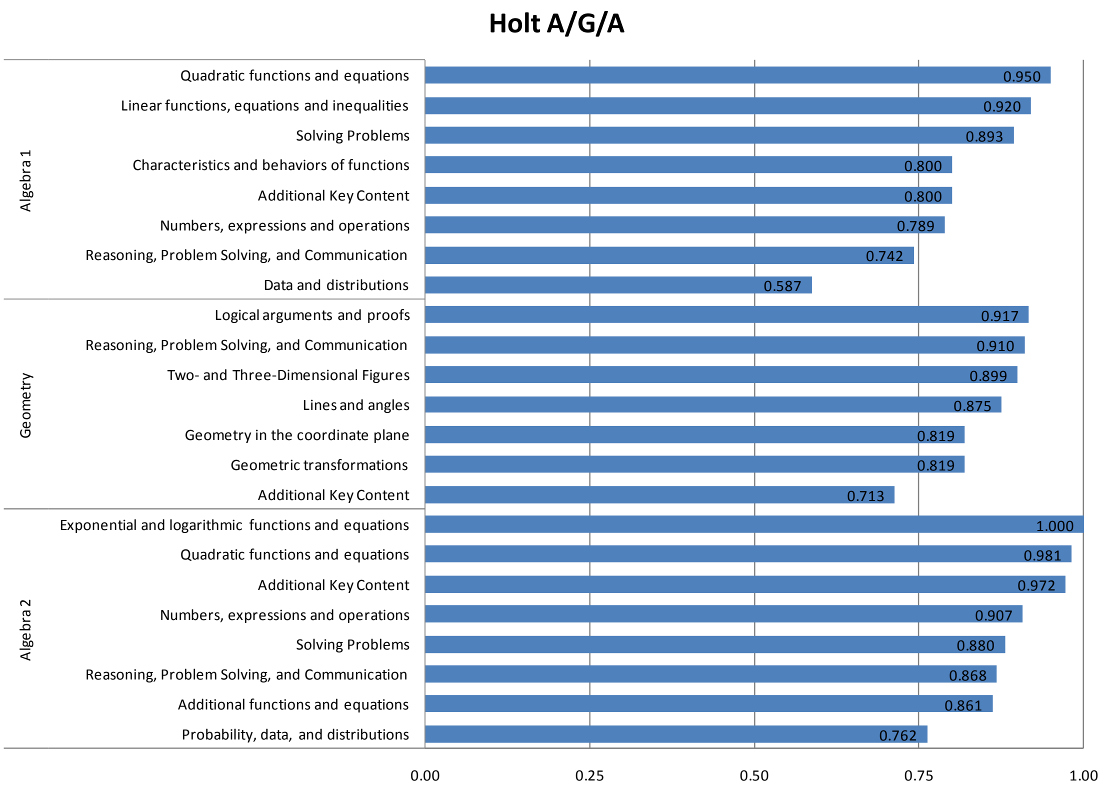
	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.759	0.854	0.785	0.861	0.839	0.823
Geometry	0.722	0.786	0.851	0.800	0.800	0.847
Algebra 2	0.722	0.792	0.833	0.807	0.813	0.824



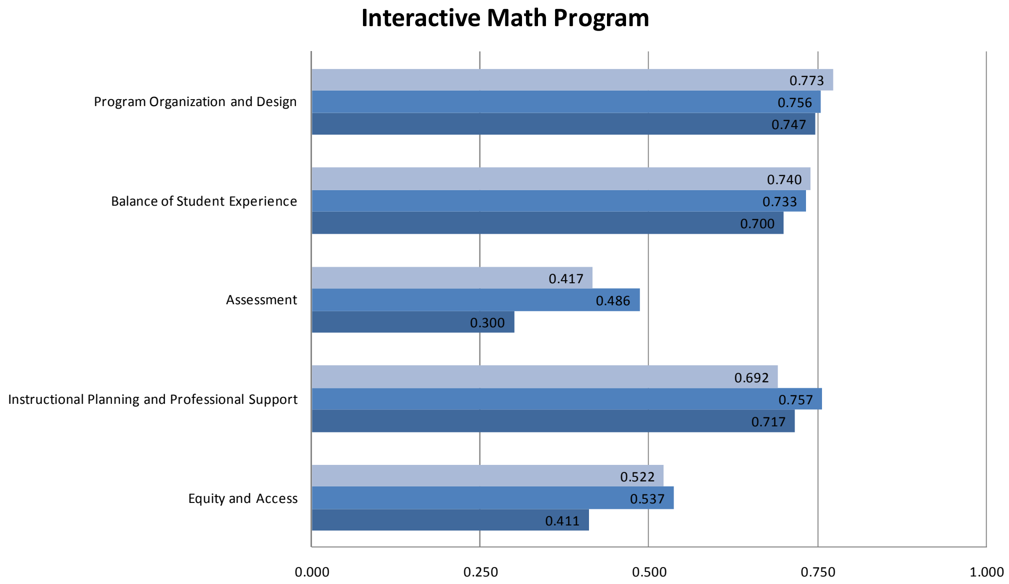
3.8.8 Holt (A/G/A)



	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.789	0.692	0.733	0.740	0.753	0.802
Geometry	0.824	0.861	0.778	0.794	0.828	0.860
Algebra 2	0.926	0.847	0.847	0.850	0.878	0.878



3.8.9 Interactive Math Program (Integrated)



	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design
Math 1	0.522	0.692	0.417	0.740	0.773
Math 2	0.537	0.757	0.486	0.733	0.756
Math 3	0.411	0.717	0.300	0.700	0.747

Figure 37. Scale results for Interactive Math Program, excluding Content/Standards Alignment.

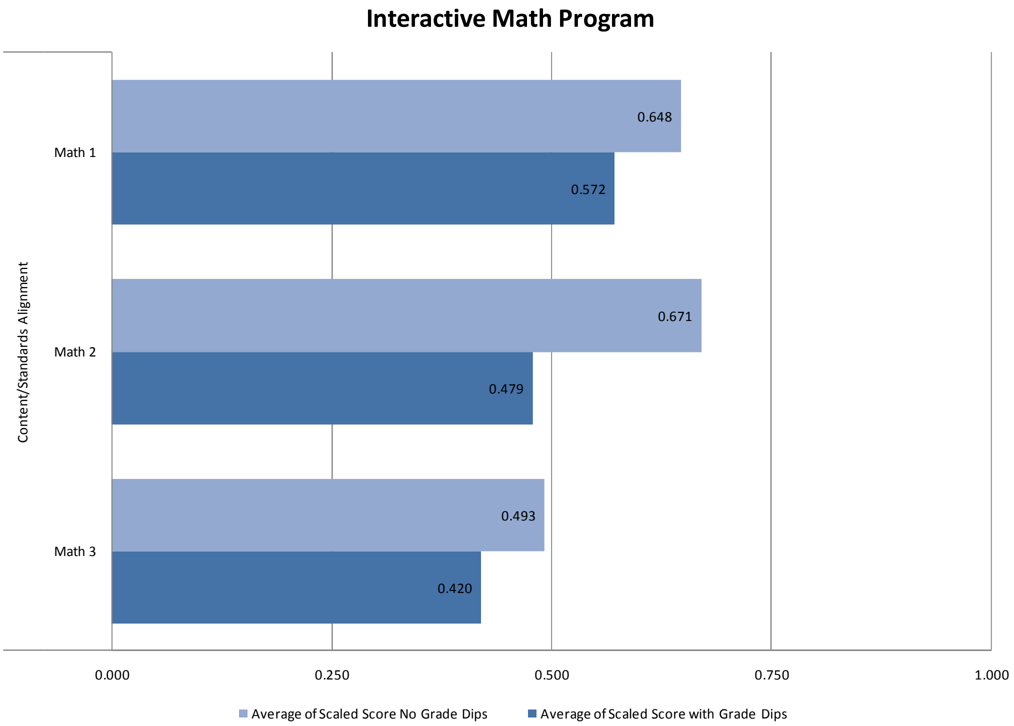


Figure 38. Content/Standards Alignment scale results, for the series as a whole (light blue) and for individual courses (dark blue).

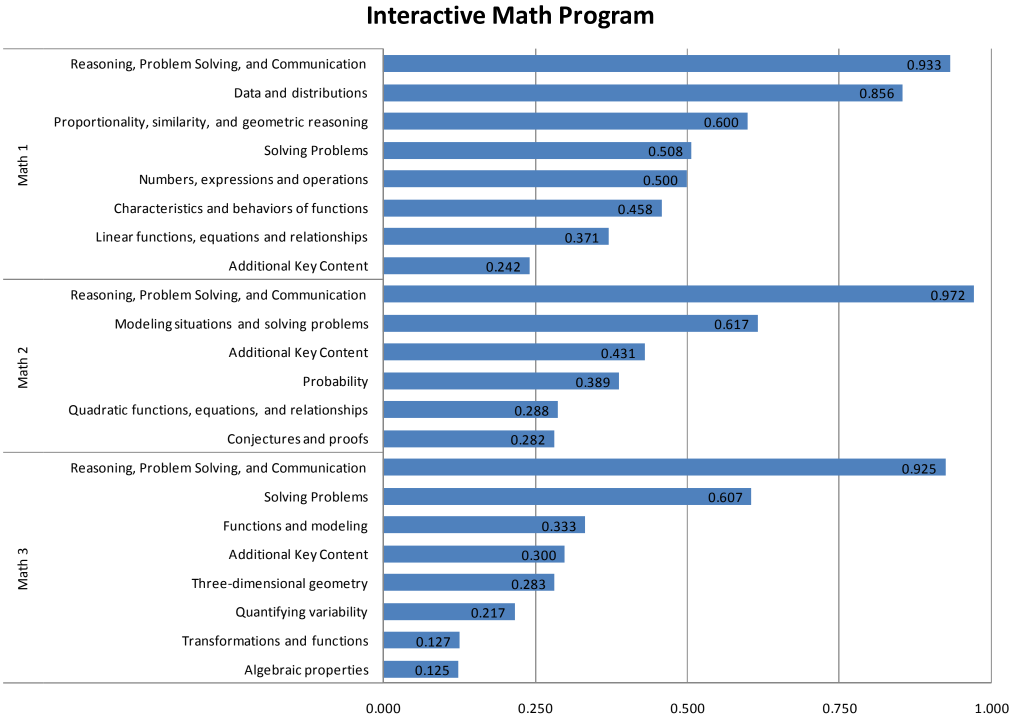


Figure 39. Core Content Area alignment results, for individual courses.

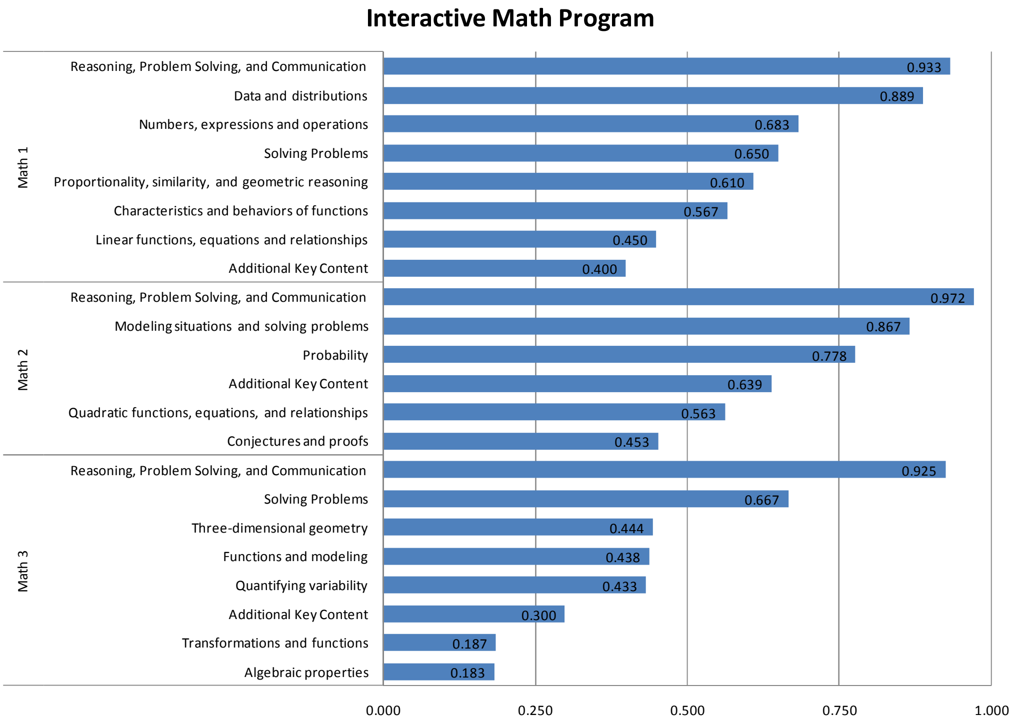
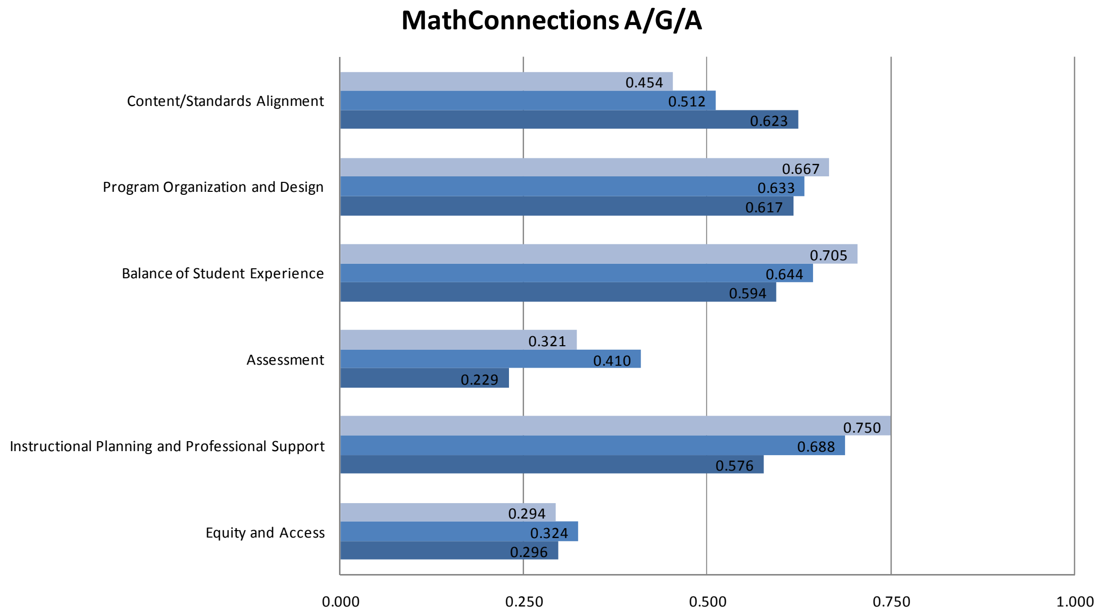
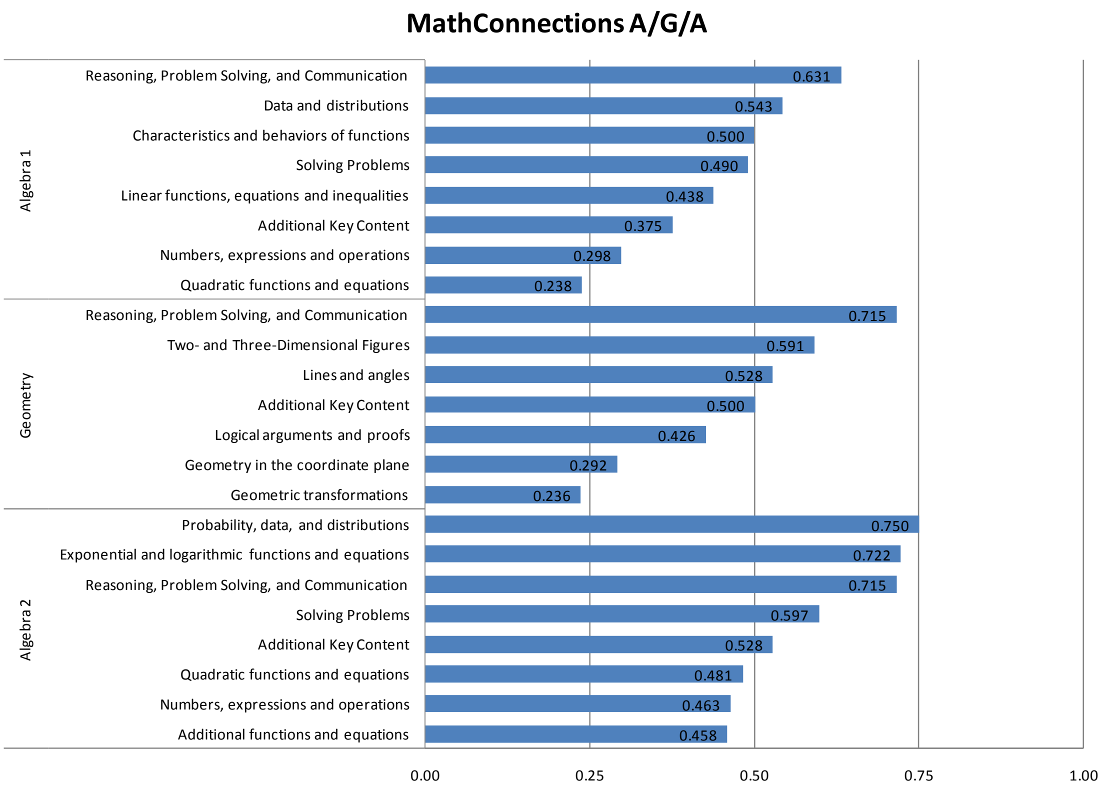


Figure 40. Core Content Area alignment results, for the series as a whole.

3.8.10 MathConnections (A/G/A)

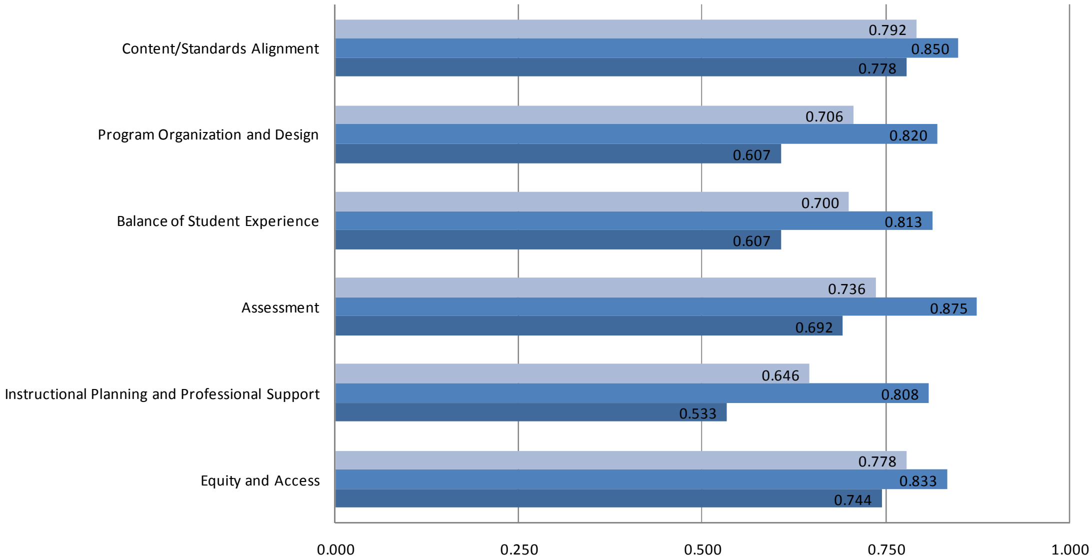


	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.294	0.750	0.321	0.705	0.667	0.454
Geometry	0.324	0.688	0.410	0.644	0.633	0.512
Algebra 2	0.296	0.576	0.229	0.594	0.617	0.623



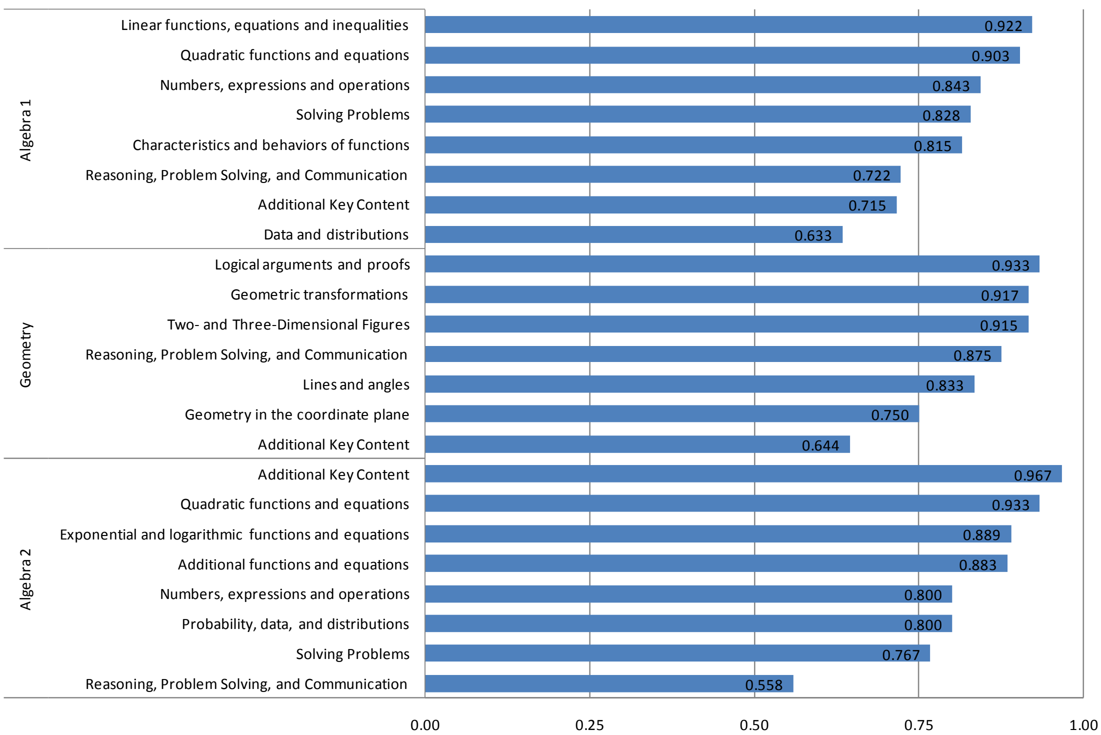
3.8.11 McDougal Little (A/G/A)

McDougal Little A/G/A



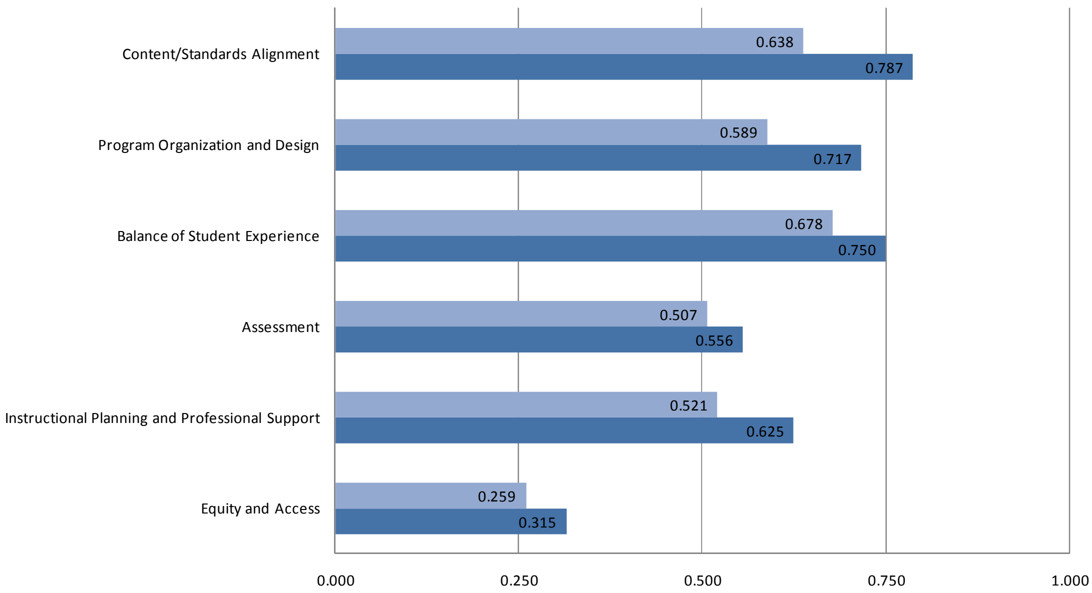
	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.778	0.646	0.736	0.700	0.706	0.792
Geometry	0.833	0.808	0.875	0.813	0.820	0.850
Algebra 2	0.744	0.533	0.692	0.607	0.607	0.778

McDougal Little A/G/A



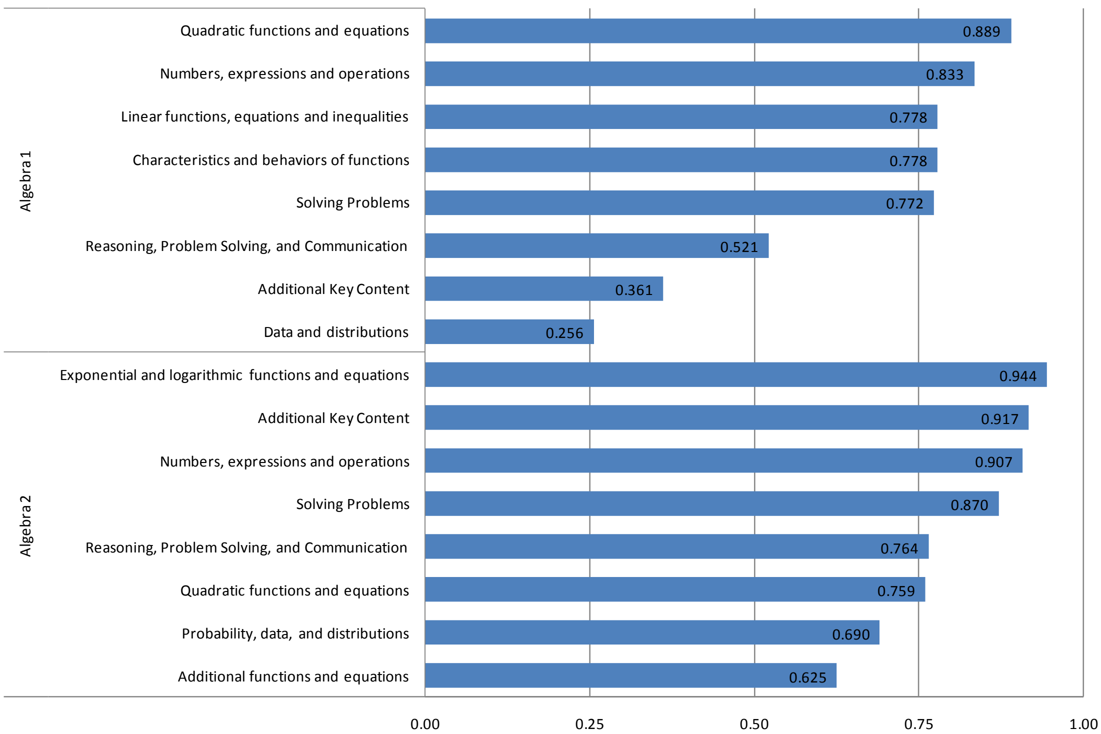
3.8.12 PH Classics Foerster (Algebra 1 and 2)

PH Classics (Foerster) Algebra



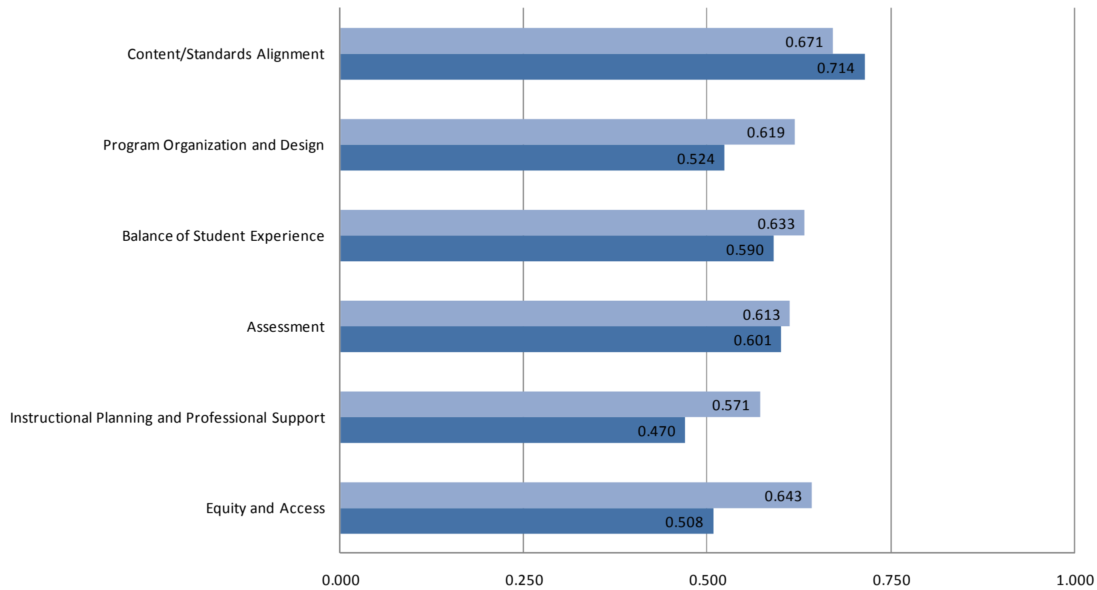
	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.259	0.521	0.507	0.678	0.589	0.638
Algebra 2	0.315	0.625	0.556	0.750	0.717	0.787

PH Classics (Foerster) Algebra



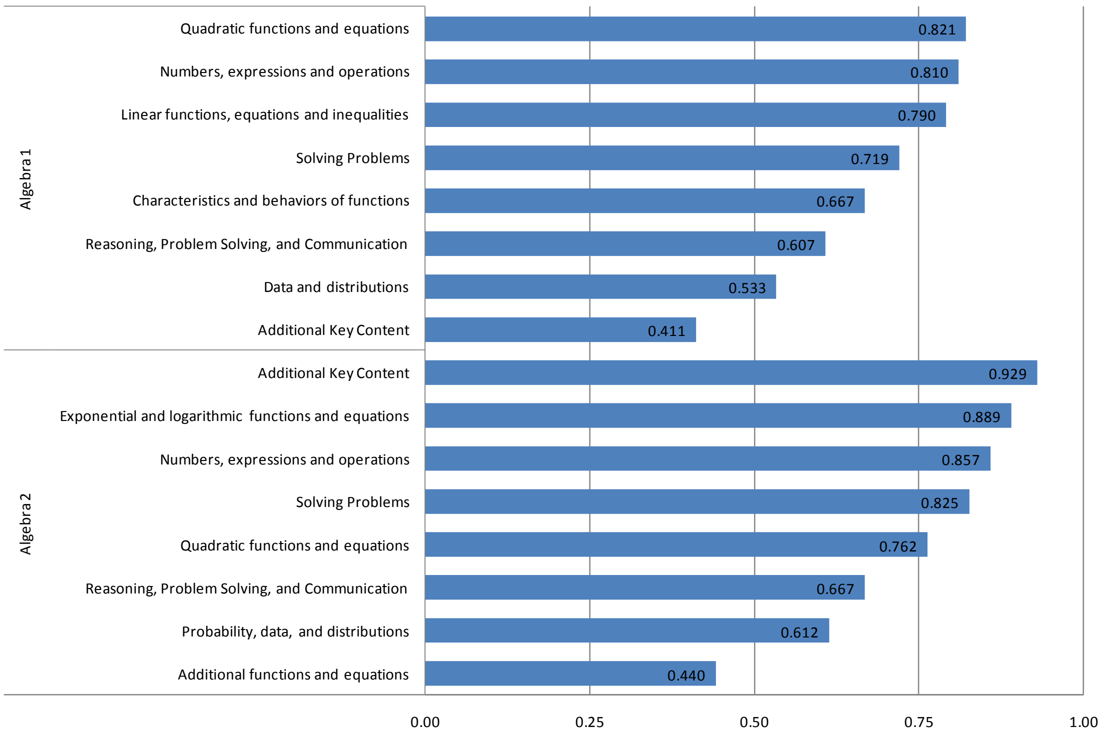
3.8.13 PH Classics Smith (Algebra 1 and 2)

PH Classics (Smith) Algebra



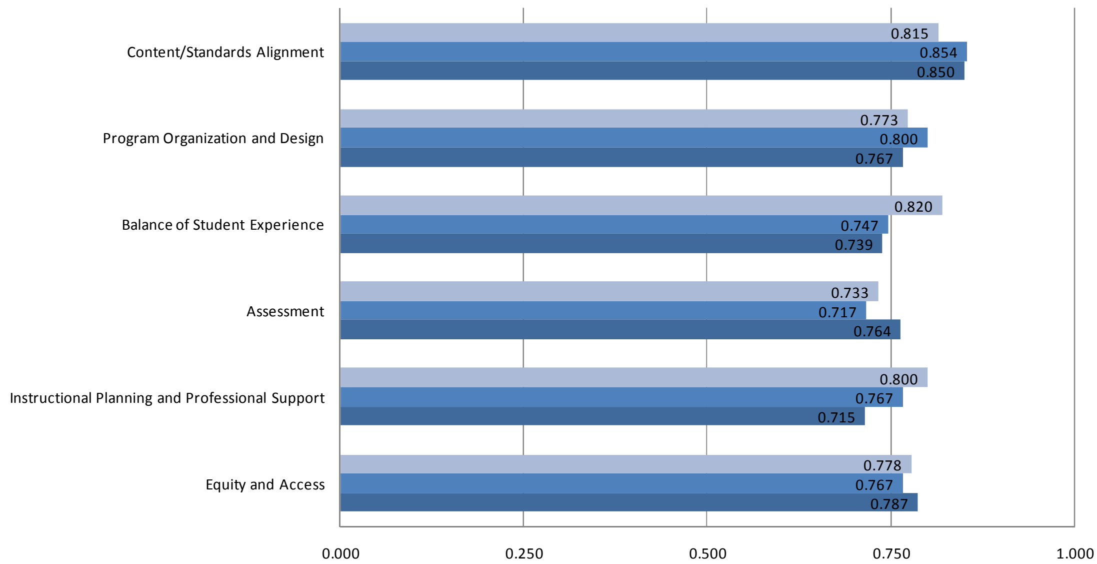
	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.643	0.571	0.613	0.633	0.619	0.671
Algebra 2	0.508	0.470	0.601	0.590	0.524	0.714

PH Classics (Smith) Algebra



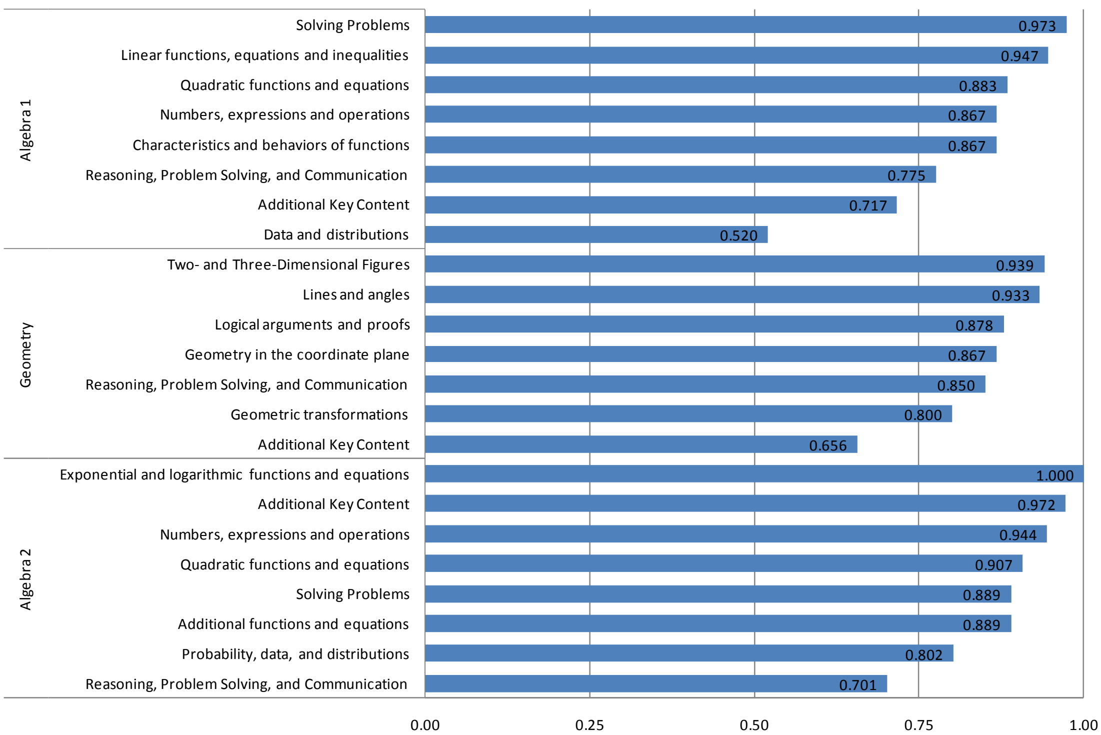
3.8.14 Prentice Hall Math (A/G/A)

PH Math A/G/A

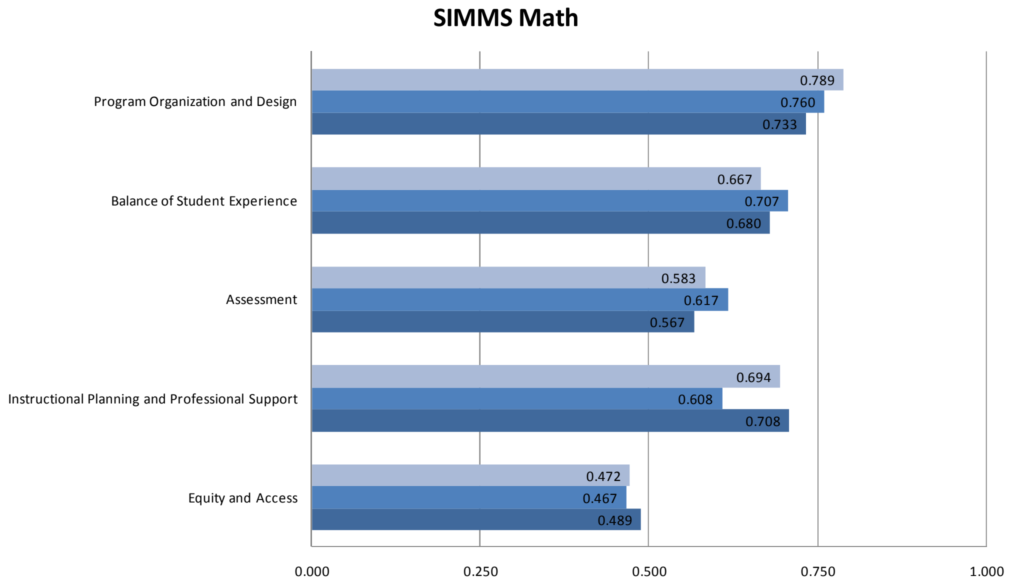


	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design	Content/Standards Alignment
Algebra 1	0.778	0.800	0.733	0.820	0.773	0.815
Geometry	0.767	0.767	0.717	0.747	0.800	0.854
Algebra 2	0.787	0.715	0.764	0.739	0.767	0.850

PH Math A/G/A



3.8.15 SIMMS Math (Integrated)



	Equity and Access	Instructional Planning and Professional Support	Assessment	Balance of Student Experience	Program Organization and Design
Math 1	0.472	0.694	0.583	0.667	0.789
Math 2	0.467	0.608	0.617	0.707	0.760
Math 3	0.489	0.708	0.567	0.680	0.733

Figure 41. All scale results for SIMMS Math, with the exception of Content/Standards Alignment.

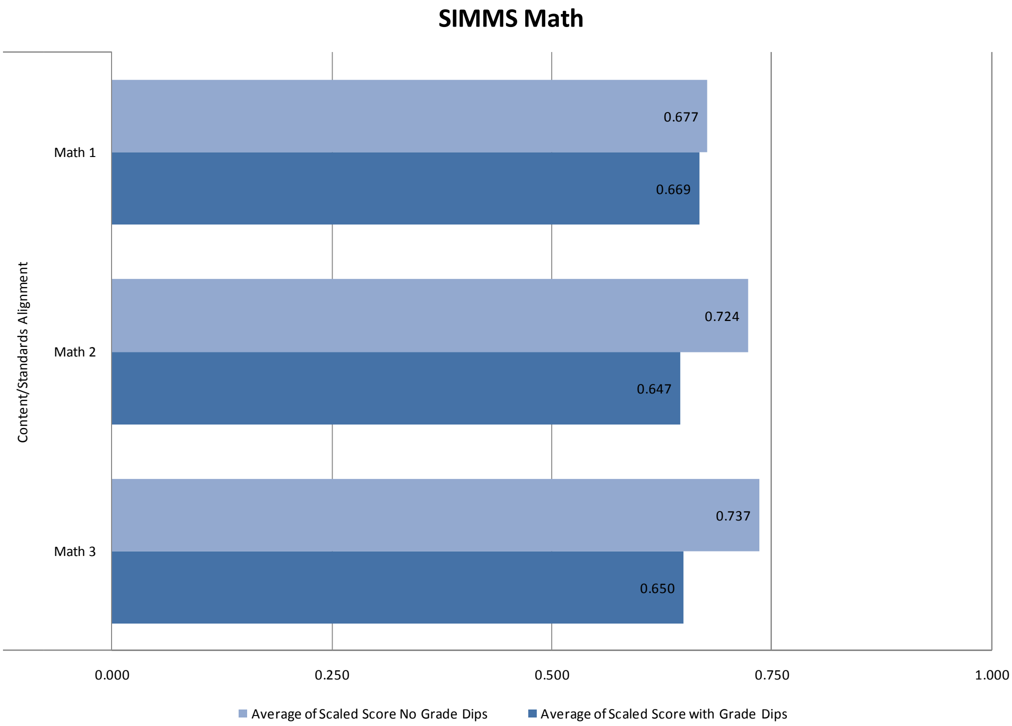


Figure 42. Content/Standards Alignment results, with and without grade dip adjustments.

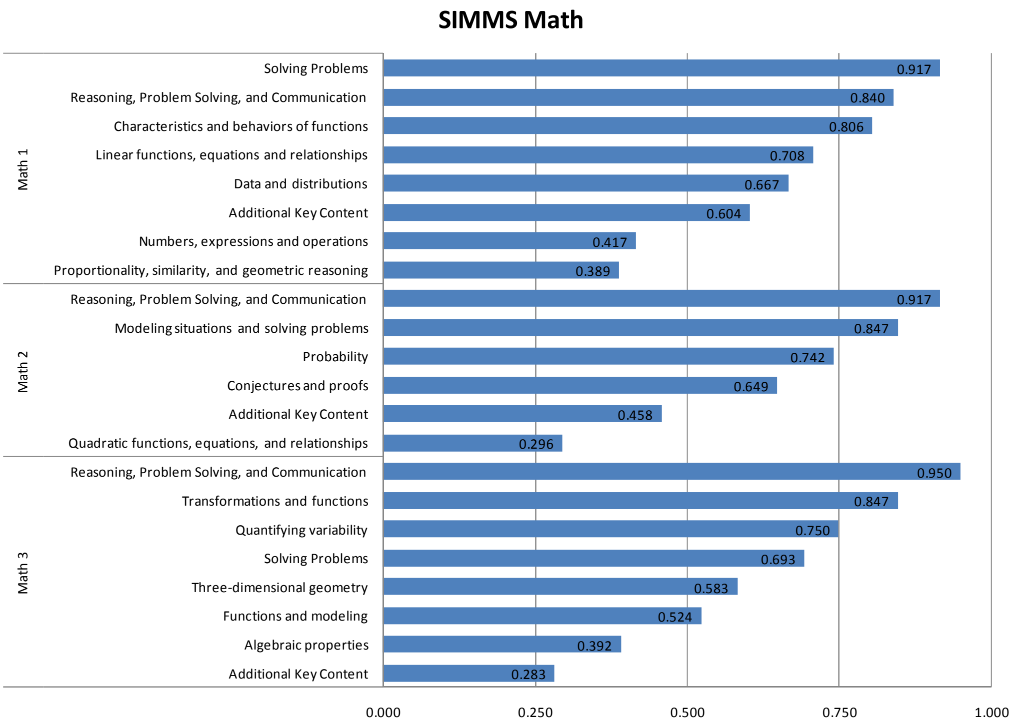


Figure 43. Core Content Area alignment results, with grade dip adjustments.

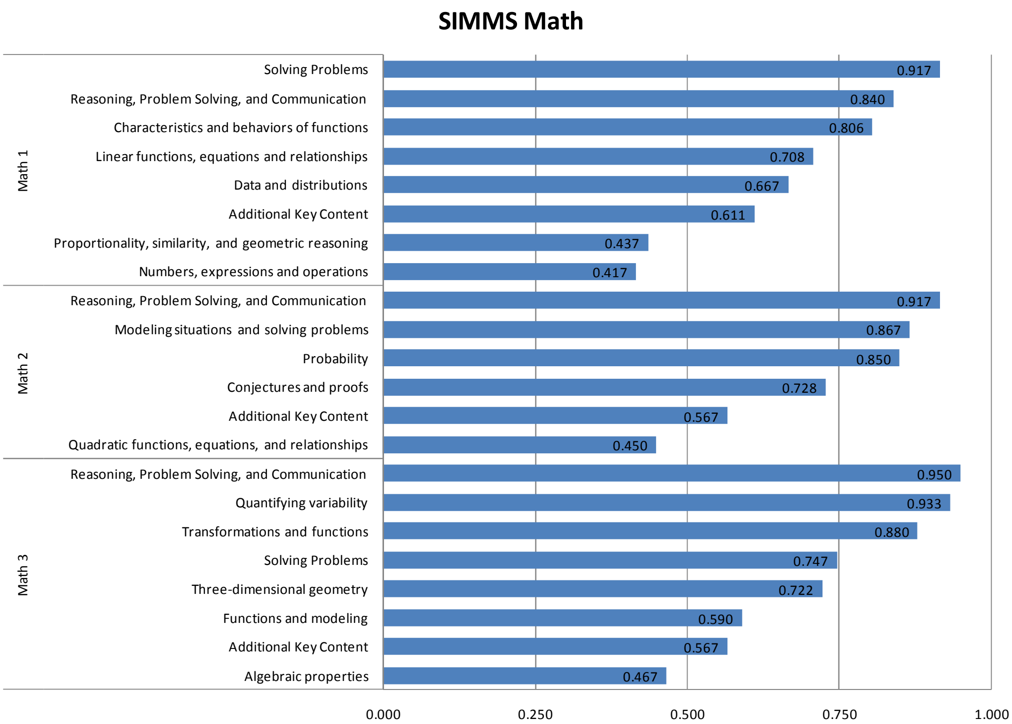


Figure 44. Core Content Area alignment results, without grade dip adjustments.

4 Mathematical Analysis of Top-Ranked Programs

The following section represents the work of Drs. King and Bright in reviewing the mathematical soundness of the top four high school curricular materials for Algebra 1 and 2, Geometry and Integrated Mathematics 1, 2 and 3. The team selected key standards that represent important development of mathematical concepts that allow students to be well-prepared to continue in mathematics study. The selection of these standards does not imply that these are more valuable than others; it simply provided a method for deep analysis on central themes.

Review of Mathematical Soundness of High School Curriculum Materials

James R. King, Ph.D. and George W. Bright, Ph.D.

The OSPI alignment study of high school curriculum materials was organized in three categories: Algebra 1/Algebra 2 materials, Geometry materials, and Integrated Mathematics materials. This review of mathematical soundness is organized in the same way.

For each category, the Performance Expectations that drove the review are listed first. However, we did not replicate the alignment study that OSPI has already completed. Rather, we looked for evidence of mathematical soundness; that is, mathematical correctness and coherent development of ideas. Only the best-aligned materials (based on preliminary analysis of the OSPI alignment study) were reviewed; the order of these reviews reflects the order of these materials in the preliminary data analysis. A summary/synthesis of the reviews is provided at the end of each section.

Any review of mathematical soundness of necessity reflects reviewers' views about mathematics itself about how an idea is, or should be, explained. Different mathematicians will potentially have different views on the "best way" to present an idea so that it is clear. Geometers and topologists, for example, "see" mathematical ideas differently, even though they study some of the same mathematical objects. No review is likely to represent all possible views. We were looking for evidence that materials provided opportunities for students to develop mathematical understanding that would be rich and deep, as opposed to compartmentalized.

In general, the materials we reviewed were found to be mathematically sound. However, we found differences among the materials related to the development of rich, deep mathematical understanding. These differences might be important to districts as they consider choosing materials for instructional use.

4.1 Algebra 1/Algebra 2

One of the major organizing ideas in algebra is *functions*. Students in Algebra 1/Algebra 2 are expected to become very familiar with linear, quadratic, and exponential functions and to gain some experience with other kinds of functions. There are many ways that the mathematics ideas related to functions might be examined. We have chosen two categories of ideas.

First, we chose to examine the development of one class of functions. The class of functions that seems most extensively developed in the high school PEs is quadratics; this is an important class of functions for high school students, both for developing mathematical maturity and in terms of application to science. The relevant PEs are listed below.

A1.1.D (M2.1.B) Solve problems that can be represented by quadratic functions and equations.
A1.5.A (M2.2.A) Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.
A1.5.B (M2.2.B) Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the x-intercepts as solutions to a quadratic equation.
A1.5.C (M2.2.D) Solve quadratic equations that can be factored as (ax + b)(cx + d) where a, b, c, and d are integers.
A1.5.D (M2.2.F) Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.
A2.3.A (M2.2.C) Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.
A2.3.B (M2.2.E) Determine the number and nature of the roots of a quadratic function.
A2.3.C (M2.2.G) Solve quadratic equations and inequalities, including equations with complex roots.

To a lesser extent, we also examined how some general ideas related to function were developed. Understanding domain/range, developing skill at moving among representations of functions, and identifying the role that parameters play are all important ideas. The Performance Expectations (PEs) below provide focus for these ideas.

A1.3.A (M1.2.A) Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.
A1.3.B (M1.2.B) Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.
A1.4.E (M1.3.B) Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.
A1.5.B (M2.2.B) Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the x-intercepts as solutions to a quadratic equation.
A1.7.A (M1.7.A) Sketch the graph for an exponential function of the form $y = ab^n$ *where n is an integer, describe the effects that changes in the parameters a and b have on the graph, and answer questions that arise in situations modeled by exponential functions.*

4.1.1 Discovering Algebra/Discovering Advanced Algebra

In *Discovering Algebra*, significant groundwork for the study of functions is laid in Chapter 7. It is significant that the ideas are developed here for functions in general; this creates a coherent mathematical sequence that is critical for helping students "see" the mathematical big picture. Domain and range for relations and functions are introduced in Lesson 7.1 and reinforced throughout the chapter. The vertical line test is introduced in Lesson 7.2, with application to the graphs of a wide range of functions/relations. Lessons 7.3 and 7.4 develop critical understanding of how functions can be used to represent different contexts; this helps motivate the need to study special kinds of functions, beginning in Lesson 7.5 (absolute value function) and Lesson 7.6 (parabolas).

Chapter 8 (Transformations of Functions) provides general background on how different function rules (e.g., $y = |x|$ and $y = |x| + 3$ or $y = x^2$ and $y = x^2 + 3$) generate graphs that look the same but are in different positions through translation, reflection, and scaling. Dealing with these issues in general prevents the need to deal with a collection of special cases when quadratic functions are studied (Chapter 9). This approach provides coherence to the mathematics ideas and would seem to make the mathematics more easily learned. For example, when students encounter Chapter 9, they will already know the effect of changing the value of a in the equation, $y = ax + b$.

Chapter 9 deals with quadratic functions. The introduction is through the modeling of real-world situations, but more standard ideas are addressed almost immediately: roots and vertex (Lesson 9.2), vertex and general form (Lesson 9.3), factoring (Lesson 9.4), completing the square (Lesson 9.6), and quadratic formula (Lesson 9.7). The extension to cubic equations (Lesson 9.8) provides a "non-example" that helps cement understanding of properties of quadratic functions. The development of critical ideas earlier in the context of many different functions should help students develop rich cognitive understanding that can be retained permanently.

In *Discovering Advanced Algebra* functions and transformations of functions are addressed in Chapter 4; again, the ideas are applied to a range of functions as a means of illustrating the power of these ideas. Lesson 4.4 specifically addresses transformations of quadratic functions. Chapter 7 (Quadratic and Other Polynomial Functions) provides specific review and extension of the study of quadratic functions. Topics include finite differences (Lesson 7.1), equivalent forms/rules (Lesson 7.2), completing the square (Lesson 7.3), quadratic formula (Lesson 7.4), and complex numbers (Lesson 7.5) which allows factoring of previously "unfactorable" quadratic expressions. Extension to higher-order polynomials provides a contrast quadratic functions; having examples and non-examples of the relevant ideas is important for helping students generalize accurately.

In general, the "Discovering" series strikes a very good balance between teaching general concepts/skills (e.g., transformations of functions) and specific concepts/skills related to quadratic functions (e.g., equation of the line of symmetry of a parabola). The mathematics is developed coherently (and soundly). By the end of the Advanced Algebra course, students should be quite ready to move on to pre-calculus.

4.1.2 Holt Algebra 1/Algebra 2

In *Algebra 1,* functions as rules are introduced in Chapter 1, but the ideas are not developed until Chapter 4. Operations on polynomials, factoring, and quadratic functions are addressed in Chapters 7, 8, and 9.

In Chapter 4, graphs are used to represent situations. Then the standard characteristics of functions are discussed: relations and functions (Lesson 4-2), vertical line test (Lab Lesson 4-2), function rules (Lesson 4-3), graphing (Lesson 4-4), and multiple representations of functions (Technology Lab Lesson 4-4). These ideas are treated somewhat compartmentally, however.

The second half of Chapter 7 addresses addition, subtraction, and multiplication of polynomials, including special products of binomials (i.e., squares of binomials and product of sum and difference of two quantities). Algebra tiles are used to model the ideas, but symbolic manipulation (including FOIL) is the technique used in the worked-out examples in the lessons.

Chapter 8 addresses factoring, first for monomials and then of general trinomials (i.e., $x^2 + bx + c$ and $ax^2 + bx + c$), with special products (e.g., difference of two squares) following. In worked-out examples, factoring is completed by identifying combinations of the factors of *c* and *a* to generate *b*. The modeling with algebra tiles in the introductory Lab Lesson is not extended into the "regular" lessons. Lesson 8-6 brings all of the techniques together by discussing "choosing a factoring method;" this is a nice way to help students reflect on what they have learned in the chapter.

Chapter 9 deals with quadratic functions. In Lesson 9-1 the idea of constant second differences is introduced and related to constant first differences already developed for linear functions. Lab Lesson 9-2 provides an opportunity for explorations leading to the equation for the axis of symmetry. Additional worked-out examples highlight relationships among the zeros, the axis of symmetry, and the vertex; graphing of parabolas (Lesson 9-3) is centered around these relationships. Families of quadratic functions (Lab Lesson 9-4) and transformations (Lesson 9-4) build on the ideas developed about graphing. The second half of the chapter deals with solving quadratic equations, completing the square, and the quadratic formula.

In *Algebra 2* functions are reviewed and extended in Chapter 1; this includes attention to transformations of functions and an emphasis on "parent" functions. Chapter 5 (Quadratic Functions) begins from this orientation of parent functions and leads to the vertex form of the quadratic equation. This is a very nice way to provide conceptual grounding for the entire chapter. Lab Lesson 5-3 connects the graph of a quadratic and the graphs of the factors of the quadratic expression; this, too, provides very good conceptual underpinning for understanding characteristics of quadratic functions. The primary extension for the remainder of this chapter is complex numbers, with applications to solving quadratic equations with no real roots.

Although the sequence of ideas in this series is fairly traditional, opportunity is provided for students to make connections among the ideas. It seems likely that students will exit with a rich understanding of the mathematics ideas underlying quadratic functions. Mathematical soundness, thus, is clearly evident.

4.1.3 Glencoe/McGraw Hill Algebra 1/Algebra 2

Relations and functions are introduced in Chapter 1, but quadratic functions are not addressed directly until Chapters 7-9. The time lag (Chapters 2-6 deal with linear equations, functions and inequalities.) might make it necessary essentially to re-teach the generic ideas at that time.

Chapter 7 deals with operations on polynomials. This is mainly a skills chapter; the word problems included seem somewhat forced. There are many exercises in each lesson (e.g., 89 exercises for lesson 7-2); it is not clear why so many similar exercises are needed. The use of algebra tiles to model operations is very nice; this sets the stage for use of this representation in Chapter 8 for factoring of trinomials. This model is explicitly tied to both horizontal and vertical symbolic recording processes for the operations on polynomials. One concern here is that students will not have much motivation to learn the skills, so they may try to memorize (rather than learn) the skills. The sequencing of the lessons and the presentation of the mathematics would seem to encourage this approach. Providing a rationale for learning this material would be a welcome addition.

Chapter 8 deals with factoring and solving quadratic equations. Again, this material is approached mainly as a sequence of skills, rather than with some underlying conceptual underpinning. Ideas addressed include factoring monomials (Lesson 8.1), factoring using the distributive property (Lesson 8.2), and factoring trinomials (Lesson 8.3). It is important that general trinomials (i.e., $ax^2 + bx + c$) are addressed first, initially through the model provided by algebra tiles. Differences of square and perfect squares are presented as special cases of the general case. This seems to be a good approach, since it puts the emphasis correctly on general ideas.

Chapter 9 deals with quadratic and exponential functions, though more emphasis is given to quadratic functions here. Lesson 9-1 introduces graphs of quadratic functions and simply states “facts” about quadratic functions (e.g., the axis of symmetry is $x = -(b/2a)$), without providing a clear rationale for why these facts are true. This approach would seem to encourage students to memorize information rather than trying to understand that information. Subsequent topics include solving by graphing (Lesson 9-2), transformations (lesson 9-3), completing the square (Lesson 9-4), and quadratic formula (Lesson 9-5). Lessons 9-6 through 9-9 provide experience with exponential functions and finite differences. As in earlier chapters, there are many exercises (e.g., 95 for Lesson 9-1), without any obvious reason for so many.

The sequencing of ideas in this Algebra 1 book is quite traditional. There seems to be an over-emphasis on skill development rather than conceptual development. However, this approach lends itself to a relatively close alignment of the book to almost any set of standards. The sequence of lessons would be understandable to most high school mathematics teachers, even though it might not generate a coherent “view” of mathematics ideas among novices (i.e., students).

Algebra 2 addresses quadratic functions mainly in Chapter 5. The work from *Algebra 1* is revisited, with extensions of some work to complex numbers. In this course, too, some key facts (e.g., “A quadratic equation can have one, two, or no real solutions.” p. 260) are simply stated, without any rationale, other than examples, for why those facts are true. If teachers do not emphasize the examples adequately, this approach would seem to encourage memorization. The development of transformations of quadratic functions is done more completely here than in the earlier book.

Chapter 6 addresses operations (including division) on polynomials, and polynomial functions. This work goes beyond that required by the Algebra 2 Standards, but it is organized to help students gain insight into an important set of mathematical ideas (e.g, rational zero theorem). This seems to be a nice extension of work with quadratic functions. Lesson 10-2 also deals with parabolas as part of the study of conic sections.

Overall, the mathematics is sound, though there is probably not enough rationale provided for helping students *want* to learn the mathematics. The approach is heavily oriented toward skill development.

4.1.4 Prentice Hall Algebra1/Algebra 2

In *Algebra 1* the concept of function is introduced in chapter 1, along with domain and range. This lays general background for later work, even though there is not much development here.

Functions reappear in much more depth in Chapter 5, which is a general discussion of functions. First, functions are used as models for events (Lesson 5-1). This is followed by relations and functions (Lesson 5-2), rules, tables, and graphs (Lesson 5-3), and four lessons on writing and using function rules. These four lessons seem to present the mathematics as compartmentalized ideas, somewhat disjoint from each other. There is no apparent underlying common thread that ties the ideas together.

Chapter 9 is focused on operations on polynomials and factoring. Algebra tiles are used as a model for multiplication of binomials, with connections made to both vertical and horizontal recording schemes. Factoring is introduced first for $x^2 + bx + c$ (i.e., finding factors of C whose sum is b; Lesson 9-5) and then $ax^2 + bx + c$ (i.e., “reverse application of FOIL”; Lesson 9-6). Special cases of difference of two squares and perfect squares (Lesson 9-7) are presented through rules as well as examples. Algebra tiles are used in an activity lab, but do not appear as part of the primary focus on instruction.

Chapter 10 begins with graphing of special cases of quadratic functions (Lessons 10.1), namely, $y = ax^2$ and $y = ax^2 + c$. Then the general case is presented (Lessons 10.2), along with graphing of inequalities. It is not clear why the special cases need to be presented first. There is a short demonstration that attempts to justify the equation of the axis symmetry. In Lesson 10-3 quadratic equations are solved by graphing, along with use of square roots to solve $ax^2 + c$, but these strategies are not connected in any way. Lesson 10-4 is factoring to solve quadratic equations, followed by completing the square (Lesson 10-5), quadratic formula (Lesson 10-6), discriminant (Lesson 10-7), and modeling (Lesson 10-8). Instruction is through worked-out examples followed by exercises. The mathematics is correct, and the sequence would probably be comfortable to most high school mathematics teachers, but there is very little help provided for students in understanding how these ideas and skills tie together. Ideas are presented in a compartmentalized way.

In *Algebra 2*, the work is reviewed and extended. There is still a tendency to reduce ideas to a series of "cases." For example, Lesson 5-4 on factoring has worked-out examples for several cases: (1) $ac > 0$ and $b > 0$, (2) $ac > 0$ and $b < 0$, (3) $ac < 0$, (4) $a \neq 1$ and $ac > 0$, and (5) $a \neq 1$ and $ac < 0$. This could clearly create the impression that identifying what case "applies" is the first step in determining how to factor a trinomial, followed by applying some memorized procedures for that case. This makes the issue of factoring an overwhelming learning burden. The major extension in this chapter is work with complex numbers, so that completing the square and quadratic formula work can include imaginary solutions.

Overall, the mathematics is sound, though there is not enough rationale provided for helping students *want* to learn the mathematics. The sequencing of examples and procedures tends to create an impression that there are many distinct "cases" that students should remember. There is too little attempt to "combine" cases under some general umbrella so that students understand how the cases are related to each other.

4.1.5 Conclusions: Algebra 1/Algebra 2

All four series provide coverage of mathematically sound content. The Discovering series and the Holt series seem to be the ones that tie together key mathematics ideas best. Since coherence of mathematics ideas is a part of mathematical soundness, these two series rate high. The Glencoe and Prentice Hall series leave an impression of compartmentalization of ideas. These two series rate somewhat lower, though they are still mathematically sound. Teachers might have to work harder to ensure that students develop deep understanding.

4.2 Geometry

One of the major themes in the Geometry standards is proof. It is clearly important to develop the idea of proof rigorously. One other major theme in Geometry is continued development of properties of figures. We have chosen to focus on parallel/perpendicular lines and parallelograms. The relevant Performance Expectations are listed below.

G.1.A (M1.4.A) Distinguish between inductive and deductive reasoning.
G.1.B (M1.4.B) Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.
G.1.C (M1.4.C and M2.3.A) Use deductive reasoning to prove that a valid geometric statement is true.
G.1.D (M2.3.C) Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.
G.1.E (M2.3.B) Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.
G.1.F (M2.3.D) Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.
G.2.A (M1.4.E) Know, prove, and apply theorems about parallel and perpendicular lines.
G.2.B (M1.4.F) Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.
G.2.C (M1.4.G) Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.
G.3.F (M2.3.J) Know, prove, and apply basic theorems about parallelograms.
G.3.G (M2.3.K) Know, prove, and apply theorems about properties of quadrilaterals and other polygons.
G.4.A (M1.3.H) Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.
G.4.B (M2.3.L) Determine the coordinates of a point that is described geometrically.
G.4.C (M2.3.M) Verify and apply properties of triangles and quadrilaterals in the coordinate plane.

What is called for is a set of theorems stating properties of parallelograms. What is needed for this are the basic theorems about angles formed by parallels and a transversal, along with the angle sum theorem for polygons and some congruence theorems for triangles. In the reviews that follow, these topics will be referred to as the standard parallelogram theorems.

4.2.1 Holt Geometry

Chapter 2 contains an extensive development of inductive and deductive reasoning, including formal rules of logic. Section 2.1 introduces inductive reasoning and conjecturing in mathematics, science, and life outside science. Next come Venn diagrams and Section 2.2 on conditional (if-then) statements. Section 2.3 addresses deductive reasoning as a way to verify conjectures. Section 2.4 is devoted to bi-conditional statements and definitions. Section 2.5 addresses algebraic proof, and Sections 2.6 and 2.7 begin geometric proof - two-column and then flowchart and paragraph proofs. All sections include a generous selection of examples and problems from geometry, other areas of mathematics, and daily life. Various strategies and representations are presented to support understanding and applications of these ideas. These rules of logic and proof are used to develop geometry topics in the rest of the book.

Chapter 3 focuses on parallel and perpendicular lines. Section 3.1 provides definitions of parallel and perpendicular lines, as well as skew lines and parallel planes. This is followed by an informal introduction to examples of parallel lines (e.g., the edges of a box). Terminology is developed here for the four pairs of angles formed by two lines and a transversal line. Section 3.2 begins with a postulate (Postulate 3-2-1) that states the equality of corresponding angles in a figure formed of two parallel lines and a transversal. Then the consequences are stated and proved as examples or problems. Section 3.3 includes a new postulate (Postulate 3-3-1) that is the converse of Postulate 3-2-1; that is, sufficient conditions that two lines be parallel. This postulate is used to prove theorems establishing that certain lines are parallel, including the case of two lines perpendicular to the same line. Section 3.3 ends with a Geometry Lab with constructions for parallel lines by compass and straightedge and by paper folding.

Section 3.4 is devoted to perpendicular lines, including some theorems about perpendicular transversals and compass and straightedge construction of the perpendicular bisector of a segment. There it is also a statement that the shortest segment from a point to a line is the perpendicular segment (the proof will come later). The Geometry Lab introduces constructions of perpendicular lines. Sections 3.5 and 3.6 deal with lines in the coordinate plane. Intersections of lines are found by solving linear equations; the concept of slope is developed and it is asserted as a theorem that parallel lines have the same slope and that perpendicular lines have slopes whose product is -1. The relationships between slope and parallelism are neither proved nor justified informally.

This chapter does a thorough job of stating and proving the basic angle theorems about parallel lines and transversals and also theorems about perpendicular lines. The inclusion of some properties of distance in the section on perpendiculars seems natural, though it does require assuming a theorem whose proof must be deferred. It is puzzling that there is no attempt to explain the slope relations for parallel and perpendicular lines, either by solving simultaneous algebraic equations or drawing simple figures with slope. This is a missed opportunity to help students make sense of the mathematics.

Chapter 6 (Parallelograms and Polygons) begins by introducing some basic definitions and theorems about polygons in general and developing the theory of parallelograms. A later part of the chapter moves on to special parallelograms and other special quadrilaterals such as isosceles trapezoids and kites. Section 6.1 defines basic terminology such as vertex, interior angle, exterior angle, and then states and proves theorems for general convex n-gons about the sum of the interior angles and the sum of the exterior angles (an important theorem that is not always given the prominence that is its due). Section 6.2 develops the standard properties of parallelograms. The properties are proved as theorems and also are studied by construction and drawing, and there are examples in the coordinate plane. Section 6.3 proves and applies conditions for parallelograms, that is, the converses of some of the theorems of 6.2. Examples and problems in the coordinate plane apply some of these theorems. Section 6.4 is about properties special parallelograms. These include parallelograms with adjacent angles equal (rectangles) and those with adjacent sides equal (rhombi). It is pointed out that squares are parallelograms with both properties. Section 6.5 proves and applies conditions for special parallelograms, including examples in the coordinate plane. The remaining sections of this chapter are devoted to other special quadrilaterals such as isosceles trapezoids and kites.

Sections 6.2 and 6.3 and Sections 6.4 and 6.5 follow a pattern of paired sections found often in this text. Certain proofs are given in the first section of the pair and then converses are developed in second section. Throughout the chapter, there are mathematically illuminating applications of parallelograms and special quadrilaterals, from carpentry to mechanical devices (e.g., car jacks).

In Summary, *Holt Geometry* includes a full treatment of what is required by the Standards and a bit more. The mathematics is developed rigorously, with proofs of theorems based on postulates. Many of the examples and exercises are either proofs of these theorems or applications of them to geometry problems. In addition there are examples of applications and some geometry lab experiments with constructions.

4.2.2 McDougal-Littell Geometry

Chapter 2 (Reasoning and Proof) begins with an extensive Section 2.1 explicitly on inductive reasoning. This features numerical and geometrical patterns and examples about data. Section 2.2 addresses conditional statements, including if-then statements and their converses, contrapositives and inverses, and the relationship between definitions and biconditional statements. Some examples address perpendicular lines and vertical angles. This section is rather short in exposition, but there are several pages of exercises. Section 2.3 is about applications of deductive reasoning, including statements of the Law of Detachment and the Law of Syllogism. Examples involve mathematics and the real world, but not much about geometry is proved in this section. An extension addresses symbolic notation, including the standard arrow notation and truth tables. Section 2.4 includes a list of postulates about the incidence relations among points, lines, and planes along with some interesting comments about how to interpret geometrical diagrams and what can be assumed in diagrams. Solution of algebraic equations is reviewed in Section 2.5. Section 2.6 (Prove Statements about Segments and Angles) includes proofs of minor results about lengths of segments and measure of angles. An example of how to write a two-column proof is provided in one example. Section 2.7 establishes standard angle pair relationships, including the congruence of right angles and the vertical angle theorem. Overall, this chapter presents the rules of logic and proof. However, the examples and illustrations seem not to go very far in addressing the difficulties inherent in understanding these concepts. The examples of proofs are technical and minor, with little geometric interest.

Section 3.1 (Identify Pairs of Lines and Angles) begins with postulates that state for a given line and a point, there is exactly one line through the point parallel to the line and one perpendicular to the line. The usual terminology is defined for pairs of angles formed by two lines and a transversal, but no theorems are proved in this section. In Section 3.2, a Corresponding Angle Postulate is stated (even though this is a really a theorem that follows from the parallel postulate in 3.1). Then three additional congruence theorems (one example and two exercises) are proved about pairs of angles defined by two parallels and a transversal. In Section 3.3 the converses of the theorems from 3.2 are proved (sufficient conditions for lines to be parallel). These theorems are used to prove the important fact that the parallel relation is transitive. Most of the exercises are immediate applications of the theorems. Sections 3.4 and 3.5 are about equations of lines. In 3.4, slope is defined and there are postulates that state if-and-only-if conditions on the slope for lines to be parallel or perpendicular. There is no indication that these properties can in fact be proved and do not need to be assumed as postulates. Section 3.6 is devoted to proving theorems about perpendicular lines. There is a proof that a linear pair of congruent angles is a pair of right angles and relates this to the real-world consequence of folding paper. Special cases of parallels and transversals when the transversal is perpendicular are spelled out. One strong feature of Chapter 3 is the explicit attention to the transitive property of parallelism. One weakness is the redundancy of assuming a parallel postulate and then assuming an equivalent statement as a postulate in the next section rather than proving it as a theorem (or at least noting that it can be done). Another weakness is the absence of any explanation or proof for the slope properties of parallels and perpendiculars, or even noting that these properties are really theorems, not postulates.

The topic of parallelograms appears rather late (Chapter 8), after a chapter on right angle trigonometry. Section 8.1 states the interior and exterior angle sum theorems for convex polygons (proofs are exercises). This is a short section with a few examples and exercises. The problem of finding the angle sum of a convex polygon is presented as a challenge but the figures supplied as hints and the answer key are incomplete in that they assume the polygon can be dissected into triangles, all of which have the same shared vertex. This teacher notes do not alert the teacher to the underlying mathematical difficulty, so the opportunity for a more challenging discussion is not supported. In Section 8.2 the usual properties of a parallelogram are stated and proved in exercises. In some problems in the coordinate plane, students are simply told that quadrilaterals are parallelograms, when students could (and should) verify this fact. Section 8.3 states the four standard necessary criteria for a quadrilateral to be a parallelogram; the opposite sides congruent theorem is proved as an example and the others are left to exercises. Here, there is a demonstration that a quadrilateral in the coordinate plane is a parallelogram by showing that one pair of sides is congruent and parallel. Students are asked to use other methods to verify that the quadrilateral is a parallelogram. An appendix to Section 8.3 is a Problem Solving Workshop that demonstrates two methods for determining whether or not a figure in the coordinate plane is a parallelogram. This is a valuable addition to the section. Section 8.4 contains if-and-only-if conditions for quadrilaterals to be rhombuses, rectangles, and squares. A Venn diagram shows how the set of squares is the intersection of the set of rhombuses and the set of rectangles. A definition of a square is given here, but rectangles and squares have been used regularly in earlier chapters (e.g., in the proofs of the Pythagorean theorem). There is no acknowledgement of the earlier appearance of squares when squares are defined in this chapter. This undercuts the presentation of geometry as an axiomatic and logical system.

The *McDougal-Littell* text covers the Washington Standards items checked in this review, but the impression of the mathematics in this text is mixed. The reasoning section seems rather shallow, though there is good discussion about how to reason from figures. The exercises routinely have examples of incorrect proofs in which students are asked to find the error. There is more attention than usual devoted to the transitive property of parallelism, and there is an extra section with explicit examples of multiple solutions of a problem. On the other hand, most of the exercises are routine or else do not really exploit the mathematical possibilities of potentially rich problems. Whether or not it is a good choice to postpone parallelograms and rectangles to the second half of the text is something that should be considered. Rectangles and squares appear informally in many earlier places in the text without any explicit efforts to reconcile the delay of rigorous development. Teachers will have to deal with possible confusion coming from this departure from logical development.

4.2.3 Glencoe McGraw-Hill Geometry

Chapter 2 addressed reasoning and proof. Section 2.1 presents inductive reasoning as using examples to form a conclusion that may – as a conjecture – lead to a prediction. Several contexts are presented, including number sequences, geometrical figures, and data. Section 2.2 introduces some aspects of formal logic including truth tables, conjunctions, and disjunctions. (The book uses this technical terminology for logical “and” and “or.”) Venn diagrams are also introduced. Section 2.3 is about conditional (if-then) statements; mathematical and real world examples are included. The converse, inverse, and contrapositive are defined, and there is a proof using truth tables showing which statements are equivalent. There is an extension about bi-conditional statements. Section 2.4 introduces deductive reasoning, including the Law of Detachment and the Law of Syllogism. An extensive set of examples is given, some of which are quite illuminating about the uses of if-then statements and possible pitfalls in understanding them. A data analysis example used to provide a contrasting example with inductive reasoning. Section 2.5 is about postulates and paragraph proofs. Some postulates about the relations among points, lines and planes are presented and then some proofs are based on these postulates. This is all correct, but the modest toolkit of postulates at this point limits the interest and challenge of what can be proved. The chapter concludes with Sections 2.6 (algebraic proof), 2.7 (proving segment relationships), and 2.8 (proving angle relationships). These sections focus on short proofs of technical and rather trivial propositions. This writing in this chapter is not a clear development of the mathematical ideas. Some helpful examples are included, but others range so far afield that they are a distraction from what is important for proof in geometry. The chapter may unintentionally communicate that the goal of proof is to find the right terminology rather than to find reasons for important mathematical statements. This seems to divert attention away from the study of geometry. In writing mathematics logically, more technical detail is not necessarily better. Focus on, and clarity about, the mathematics content being studied is essential.

Chapter 3 is devoted to parallel and perpendicular lines. Section 3.1 defines parallel and skew lines, as well as parallel planes, with exercises to find such lines in a wedge of cheese or a cubical box. Terminology about angle pairs defined by a transversal is introduced, along with practice using this terminology. Section 3.2 is about angles and parallel lines. Based on a postulate about corresponding angles, the congruence of other angle pairs is proved. The special case of a perpendicular transversal is a theorem, and there are examples and exercises about angle measures in geometry figures and in real-world examples. Section 3.3 includes postulates about the slope relationships for parallel and perpendicular lines; there are no explanations for why these are true. In Section 3.4, most of the work is finding the equations of lines through two points, but there is also an example of a line through a point that is parallel to a given line. In the Geometry Lab at the end there is a more substantial example developed, which is to find the equation of the perpendicular bisector of a segment in the coordinate plane.

Section 3.5 is about proving lines are parallel in the plane. Postulate 3.4 asserts that if two lines are cut by a transversal so that all the corresponding angles are congruent, then the lines are parallel. This is followed by a description of the construction of a line through a point parallel to a given line. Then comes Postulate 3.5, which is a version of the Euclidean parallel postulate. Next are four theorems that state the congruence of a pair of angles implies that two lines are parallel. The proofs are left to the exercises. Several aspects of the mathematical development in this section are troubling. First, Postulate 3.4 is unusual and awkward, since it is sufficient that only one pair of the corresponding angles be congruent. In fact the statement that one pair of corresponding angles is missing, though one theorem correctly asserts that if one pair of congruent alternating interior angles implies the lines are parallel. Second, there is the curious appearance of the Euclidean Parallel Postulate. It is stated that the straightedge and compass construction proves that there is at least one parallel line, but this Postulate is needed to prove that there is only one. However, the two postulates about corresponding angles already given are sufficient to prove the Euclidean Parallel Postulate, so the insertion of this additional postulate is unnecessary and confusing. Also, the historical note (i.e., Euclid needed only five postulates to prove the theorems “in his day”) is very odd.

Section 3.6 on perpendiculars and distance begins by asserting without proof that the distance from a point to a line is the length of the perpendicular segment from the point to the line. The uniqueness of the perpendicular is stated as a Postulate in the text, but the fact that the length is minimal is not justified. At the end of Section 3.6, the concept of distance between two parallel lines is introduced as the distance from any point on one line to the other line. This is followed by a detailed example in which the distance between two parallel lines in the coordinate plane is computed. This section has some logical difficulties. Early on, an alternate definition of parallel lines is given; namely, two lines are parallel if they are equidistant. Since the proof of equidistance depends on rectangle properties that are not yet developed, the definition can only be stated here without proof. If distance is going to enter into this chapter, there should at least be a coherent explanation so that it is clear that there are statements that must be proved later, so that students will not be confused about the underlying mathematics. Worse, students are asked to prove that if two lines are equidistant from a third, then the two lines are parallel. Since the logical development is deficient here, no proof could be correct. The answer in the teacher's edition is based on the coordinate plane, so there is real confusion about whether a proof is supposed to be in the Euclidean plane (no coordinates) or in the coordinate plane.

A strong point of this chapter is that after a rather lengthy review of the various forms of the equation of a line, there are some substantial applications of the algebra to constructing parallel lines and perpendicular bisectors, finding distance from a point to a line, and other applications. On the other hand, the development of angles defined by transversals introduces an unusually large number of terms for the pairs of angles; the attention necessary for mastering this terminology diverts the narrative from more important geometric content. The chapter also provides rather weak support for understanding and proving, as opposed to memorizing, these properties. It is unfortunate that the slope properties of parallels and perpendiculars are presented as postulates rather than as theorems than can be explained and proved (with algebra and at least informally with geometry). There are some exercises that call for proof, but there is little support for learning how to write proofs. And the logical flaws in the development of the parallel postulate and in the treatment of distance pointed out above detract significantly from the mathematical rigor and clarity.

Chapter 6 deals with parallelograms and polygons. Section 6.1 presents the interior and exterior angle sum formulas for a convex polygon. These formulas are considered in a number of exercises about general polygons and also previews of some special cases. In Section 6.2 the standard properties of parallelograms are stated and proved (i.e., one example of a proof, the rest as exercises). Some examples of parallelogram arms from the real world are shown. In Section 6.3 sufficient conditions for a quadrilateral to be a parallelogram are proved. Section 6.4 is about rectangles, with a proof of equal diagonals being a necessary and sufficient condition for a parallelogram to be a rectangle. Section 6.5 is about rhombi and squares, including the definitions and properties of the diagonals. This chapter develops the ideas clearly and correctly, with several examples of proofs provided as models. The inclusion of examples for the coordinate plane meets the requirements of Performance Expectation G.4.C.

The *Glencoe* text covers the topics required by the Washington Standards. In many places the treatment is clear and correct. But as noted in the section summaries, there are several instances of logical flaws, a conflation of genuine postulates and unproved theorems and some confusing mathematical statements that detract from the text.

4.2.4 Prentice-Hall Geometry

Chapter 1 lays significant groundwork for the study of geometry. Topics include informal geometry, important definitions (e.g., parallel and skew lines, parallel planes, perpendicular lines), compass and straightedge constructions, the coordinate plane (e.g., formula for the midpoint of the segment), and the distance formula (based on the Pythagorean Theorem). The text carefully distinguishes the use of the word "segment" from the word "line." Some exercises contrast circular definitions with the use of undefined terms in mathematics, and the discussion addresses the tension between the logical development of geometry as an axiomatic system and the fact that students will have already studied informal geometry in earlier grades. It attempts to make clear what is proved and what is not yet proved.

The development of logical tools for proof is taken up systematically in Chapter 2. Section 2.1 introduces conditional (if-then) statements right away, with many examples, including rewording of statements not in if-then form into if-then form. Counterexamples and converses (and the truth value of the converse) are introduced and illustrated. The chapter also includes Venn diagrams and standard arrow symbols. Section 2.2 contains a careful introduction to biconditional statements and definitions. Section 2.3 is about deduction, including the Law of Detachment and the Law of Syllogism. Examples and problems focus on the effective and correct use of these tools. Section 2.5 centers on the use of equations and algebra for solving questions in geometry. Section 2.6 uses these algebraic tools to make angle computations, including proving that vertical angles are equal. The chapter does a good job of presenting the important tools of logic and proof and addressing possible points of confusion. It is efficient in that it does not digress into a study of logic or algebra beyond what is needed for geometry.

Chapter 3 addresses parallel/perpendicular lines. Section 3.1 defines three pairs of angles formed by a transversal of any pair of lines and then moves to the case of parallel lines with the postulate that corresponding angles formed by a transversal intersecting a pair of parallel lines are congruent. The other angle relations formed by parallels and a transversal are proved. The teacher notes correctly point out that the Corresponding Angle Postulate is a variation of the Euclidean Parallel Postulate. This section is distinguished in that it moves briskly from definitions to the geometrical content of angles and parallels. Section 3.2 contains a postulate and then theorems stating the usual conditions that congruence of one pair of angles (corresponding, or alternate interior, etc.) formed by a transversal and two lines implies that the two lines are parallel. The theorems are correctly labeled as converses of the theorems in the previous section. Section 3.3 is about parallel and perpendicular lines. Perpendicular transversals are used to give a correct proof that two lines parallel to the same line are parallel. Section 3.4 proves that the sum of the angles of a triangle is 180 degrees. By proving this theorem in the chapter on parallels, the text provides an interesting and powerful application of the theory of angles and parallels. After this theorem, the exterior angle theorem is proved and classifications of triangles by angle are introduced. Section 3.5 proves angle sum theorems (both interior and exterior) for convex polygons. Sections 3.6 and 3.7 deal with the slopes of parallel and perpendicular lines. These relations are correctly presented as concepts that will be proved later rather than as postulates. Section 3.8 presents step-by-step straightedge and compass constructions of parallel and perpendicular lines. The treatment of parallels in this chapter presents the theorems about angles and parallels concisely but effectively. Distance does not appear in the section (thus avoiding some logical sequence problems). The mathematics is correct, including the appropriate distinction between logically necessary postulates and facts that are really theorems than can be proved later. Also, the understanding of the parallel postulate is correct.

Chapter 6 is about quadrilaterals, including application of the angle sum theorem for convex polygons, which was proved in Chapter 3. Section 6.1 begins with the definitions of special quadrilaterals, along with a diagram relating the logical relationships among the various kinds of quadrilaterals. Exercises develop examples and consequences of the definitions, including examples in the coordinate plane. Section 6.2 presents the standard properties of parallelograms. The equality of opposite sides is proved in a detailed proof. Included is one useful theorem that is often not stated: if three parallel lines cut off two congruent segments on one transversal, then they cut off two congruent segments on any transversal (a situation that occurs multiple times with notebook paper or street grids). Section 6.3 contains the sufficient conditions to prove that a quadrilateral is a parallelogram. Careful proofs are given of two of the theorems. Examples and investigations are included. The topic of Section 6.4 is special parallelograms, namely rhombuses and rectangles. Theorems about the diagonals are proved (i.e., necessary and sufficient conditions). Numerous exercises are included, some about problem solving and some asking for proofs. This development of the theory of parallelograms is complete and clear. The extra theorem about transversals and congruent segments is an interesting and useful application. The examples of proofs do a good job of making clear how proofs are written.

The selected topics from the Washington Standards are covered fully in *Prentice-Hall Geometry*. Some things that distinguish this text are the unusual placement of the angle sum theorems and the inclusion of an additional theorem about parallels. More importantly, the text shows good mathematical judgment. The relationship between postulates about parallels and angles and the Euclidean parallel postulate is understood correctly. The text refrains from labeling every unproved fact as a postulate, instead stating them as “principles” that are merely as-yet unproved theorems. Also, the text avoids some tricky points making hidden and unproved assumptions about distance and parallelism. There is a generous supply of exercises and activities.

4.2.5 Conclusions: Geometry

The *Mathematics Standards* state that students should know, prove, and apply theorems about angles that arise from parallel lines intersected by a transversal. The development adopted by the reviewed texts is to assume as a postulate that for any two parallel lines intersected by a transversal corresponding angles are congruent. It is immediate to prove that a number of pairs of angles are either congruent or supplementary (for example, alternating interior angles are congruent). Then, as a second postulate, the converse of the first postulate is assumed. After this, it is proved that the necessary conditions in the earlier theorems are in fact sufficient conditions.

One important “backstory” for this development is that these postulates imply the Euclidean Parallel Postulate (EPP). To be precise, the second postulate can be proved as a theorem in Euclidean geometry and the first postulate is equivalent to the EPP. Some of the textbooks try to include some of this background, more or less successfully as the reviews note. It is not strictly necessary for students to know this background for their study of geometry, but if the choice is to introduce the EPP, it would be better to tell the story correctly.

The texts differ in the accuracy and completeness with which they present the relevant mathematics. *Holt Geometry* and *Prentice-Hall Geometry* seem to be the most successful in this regard. Teachers might have to be more careful in explicating the mathematics of the other two texts.

4.3 *Integrated Mathematics*

All of the integrated mathematics materials were three-book series. The same threads were examined here as were examined in the Algebra 1/Algebra 2 and Geometry materials.

One characteristic that distinguishes integrated mathematics materials from more traditional materials is the extensive use of contexts and applications as the focus of attention. Mathematics ideas are typically not presented as "naked" mathematics, but rather as ways to solve problems. This does not mean that the mathematics is less important or less well developed, but it does make a review of mathematical soundness somewhat more complex.

4.3.1 Core-Plus Mathematics

Functions. In Course 1, quadratic functions (Unit 7) are introduced through specific examples (e.g., projectile paths). This specific approach has the potential to create "stereotypical images" in students' minds that may be difficult to overcome to create a general understanding of quadratic functions. It appears, however, that by the time students work through Investigation 3 a general understanding should have developed. The teacher's role in debriefing students' work is probably critical so that students understand how the parameters for the general quadratic function influence the shape and position of the graph.

In Course 2, quadratic functions are treated as one kind of nonlinear function (Unit 5). This is a strength mathematically, since it helps reinforce the similarities and difference among different kinds of nonlinear functions. It is in this unit that domain and range are emphasized (Lesson 1, Investigation 2) and factoring is developed (Investigation 3). The area model (i.e., algebra tiles) is used to motivate techniques for factoring. Solving of quadratic equations is developed, and the quadratic formula is presented, but it appears to be developed only in the "On Your Own" section of problems/exercises. Lesson 2 focuses on Nonlinear Systems; this provides an immediate application of what was dealt with in Lesson 1.

In Course 3, quadratic functions reappear in Unit 5, Lesson 2: Quadratic Polynomials. Completing the square is the focus of Investigation 1; by this point, all students should be intellectually prepared to understand the mathematics of this idea at a deep level. The vertex form of the equation is addressed here, and complex numbers are introduced with the obvious extension to quadratic equations with no real solutions can be examined.

Geometry. In Course 1, the study of properties of figures begins in Unit 6. "The focus here is on careful visual reasoning, not on formal proof." (Formal proof is addressed extensively in Course 3.) Unit 6 is "developed and sequenced in a manner consistent with the van Hiele levels of geometric thinking." Senk's data (1986) suggest strongly that students who attempt to study proof before the development of Level 2 thinking (e.g., Fuys, Geddes, & Tischler, 1988; Van Hiele, 1986) are unlikely to be successful.

Unit 6 is organized to help students develop Level 2 thinking. Because the study of formal proof is delayed another year, there are additional opportunities for this kind of thinking to develop.

Unit 6, Lesson 1, deals with a variety of topics at an informal level, including conditions that determine triangles or quadrilaterals (e.g., triangle inequality), angle sums for polygons, SSS/SAS/ASA properties of triangles, reasoning about shapes, and the Pythagorean Theorem. Some constructions are included as an extension of this work. Lesson 2 addresses symmetries of figures, angle sums of polygons, and tessellations. The tasks here emphasize relationships among different shapes; these help students internalize Level 2 van Hiele thinking. Specific attention is paid to interior and exterior angles of polygons. Lesson 3 deals with three-dimensional shapes. This work, too, is informal. It is much more exploratory, since students are likely to have less well-developed understanding of three-dimensional shapes.

The primary attention to geometry in Course 2 is coordinate geometry. This is important but does not relate directly to the threads being reviewed here.

In Course 3, Unit 1 addresses proof. The unit begins with an introduction to logical reasoning set in many different contexts, not just geometry. This is an obvious strength for the study of proof. Lessons 2, 3, and 4 address proof in geometry (mainly study of angles when parallel lines are cut by a transversal), algebra, and statistics. Both in this Unit and in Unit 3, the teacher notes are extensive, with considerable detail provided for each of the proofs. These notes would support teachers well in leading discussions that were effective at helping students internalize the critical mathematics ideas.

Unit 3 addressed triangle similarity (Lesson 1) and congruence (Lesson 2). In Lesson 1, students explore a variety of conjectures, for example, all isosceles right triangles are similar. There are numerous applications of similarity which provide a rationale and motivation for proofs. As one would expect in a "proof unit," there are numerous classic mathematics relationships established and proved. In Lesson 2 congruence is studied as a special case of similarity. Included are the classic triangle congruence theorems, with attention also paid to perpendicular bisectors of sides, angle bisectors, and medians. This is followed by an equally extensive study of the properties of quadrilaterals, with particular attention to parallelograms.

In summary, the mathematics in *Core Plus* is mathematically sound and very well sequenced to support student learning at a deep level.

4.3.2 SIMMS Integrated Mathematics

Functions. In Level 1, quadratic functions are addressed in Module 10. Distance/time graphs are used as a context to support comparison of these graphs to determine average velocity over a time interval, leading to linear modeling for objects moving at constant speed. Quadratic functions are introduced in Activity 3; topics include coordinates of the vertex, vertex form of quadratic function rule, families of functions (based on $y = x^2$), and translation of parabola graphs. The Chapter ends with an exploration of the quadratic modeling of data.

One difficulty in analyzing the Teacher's Guide is that there is very little discussion of the mathematics; detailed answers are provided for each task, but there is no rationale provided for the sequencing of these tasks. It might be difficult for some teachers to lead appropriate debriefing of the exercises so that students truly internalize mathematical understanding. Merely solving the tasks correctly does guarantee depth of understanding.

In Level 2, quadratics are addressed in Module 6 as part of the study of polynomials, with parabolas highlighted in Activity 2. Topics addressed include fitting a parabola to three non-collinear points, roots and factors of polynomials, and effects of changing the parameter, *a*, in the general form of a quadratic function. Embedding quadratic functions in a more general context is a strength for supporting students' understanding.

In Level 3, Module 11, transformations of functions are addressed. This is a general treatment, though some examples are quadratic functions. There does not appear to be a significant development of quadratic functions, *per se*, in Level 3.

Geometry. In Level 1, Module 1, simple ideas about angles are used to introduce techniques for studying mathematics. There is little development here. The Activities in Module 4 address surface area of three-dimensional figures, tessellations, and area of regular polygons. These ideas "feel" disconnected, with little obvious attempt to highlight common features of the ideas.

In Level 2, Modules 3 and 7 each address geometric ideas, but again the connections among them are not immediately obvious. Module 3 addresses area of regular polygons and surface area and volume of three-dimensional shapes. Module 7 addresses angles formed by a transversal of parallel lines, tangents and secants to circles, and dilations. Many teachers might need help in communicating to students what key mathematics ideas underlie the tasks. Module 12 is a more traditional treatment of proof. Three areas are addressed: Pythagorean Theorem, triangles, and quadrilaterals. However, there may not be enough tasks to support deep understanding by students of the nature of proof.

In Level 3, Module 6 is a more general treatment of proof. It is strange that this Module is after the Module in Level 2 on proof of triangle and quadrilateral theorems. Certainly students by Level 3 should be ready to learn this material, but it might also have been useful prior to the work with congruent triangles in Level 2.

In summary, the development of mathematical ideas is difficult to follow in *SIMMS*. This observation seems reinforced by examination of the alignment grid provided by the publisher. Many of the Performance Expectations are addressed in parts of problems scattered across a wide range of pages. It seems likely that some teachers might have difficult in helping students internalize the mathematical ideas based on the tasks they have completed. Also, the Modules seem too short to support in-depth development of mathematical ideas.

References

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Senk, S. L. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education*, 20(3), 309-321.

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. New York, NY: Academic Press.

5 Data Analysis Methodology

5.1 Approach

Prior to data collection, we developed an analysis plan consisting of five main steps:

1. divide the data by program type (Algebra, Geometry, Integrated Math);
2. calculate the average score on standards items;
3. compare those scores to a threshold of 0.7;
4. calculate weighted average scores across all factors for those that surpass the threshold; and
5. compare these remaining programs to determine the top 3 (or fewer).

In calculating both the standards score and overall weighted scores, we considered using a linear mixed effects model to control for possible reviewer bias by including a random intercept for reviewer. However, since the design is not complete – i.e., only some reviewers review each program – we cannot fully separate reviewer effects and program effects. Thus, if a particular reviewer happened to see only the most strongly aligned programs, their overall average score would be high, not because they were biased, but because they scored strong programs. Adjusting for this would effectively be punishing the programs that were seen by that reviewer. Thus, we chose to test for reviewer bias first, and only use the adjusted model if there was evidence of severe bias. If not a simple average or weighted average was to be used.

There are a number of legitimate ways to then compare the program scores, both to the threshold of 0.7 and to each other. We hoped to keep the analysis relatively clear and simple, to facilitate transparency of the report. To this end, we opted to use t-tests to compare programs, a widely used and well understood method. In this study, we are comparing averages of many scores for each program, which allows us to use a t-test even though the data are not normally distributed. The results, threshold tests and program comparisons, were kept to the traditional 0.05 significance level.

A significance level of 0.05 is meant to imply that we are willing to accept a 5% chance that we will reach the wrong conclusions based on the data we collect. There are theoretical results that show that this significance level is maintained when doing one or more tests (controlling for multiple comparisons in the latter case) *when the analysis plan is constructed without looking at the data*. Once analysis decisions are made based on what we see in the data itself, we no longer can make the assumptions necessary to know the distribution of outcomes. In this case, p-values no longer carry the meaning they did when we planned our analysis in advance; we cannot make rigorous conclusions about the statistical significance of a result.

5.2 Response Scales

In data collection, Content/Standards Alignment (hereafter “content”) questions were rated on a Not met/Lacking content/Lacking practice/Fully met scale. Other factors (Assessment, Equity and Access, Instructional Development and Professional Support, Program Organization and Design, and Student Learning) were rated on a 4 point Likert scale.

These are ordinal variables, and not inherently numeric. In the analysis that follows, we assume that the "distance" between two consecutive levels is the same across a scale. That is, the value added by moving from "Not met" to "Lacking content" is the same as moving from "Lacking content" to "Lacking practice" in the standards. Similarly, the value added moving from "Strongly disagree" to "Disagree" is the same as from "Disagree" to "Agree" on the Likert Scale.

The data were initially recorded on a 0-3 integer scale. For standards items, reviewers also noted whether the standard was found in the appropriate text or in an adjacent one, with half credit given for a standard met in an adjacent text. We rescaled both content and other factors scores to be on a [0,1] scale by dividing by 3.

5.3 Distributions of Scores by Course Type

The following tables show characteristics of the distribution of scores for algebra, geometry and integrated programs, respectively, broken down by the two scales, content and other factors. The unweighted average scores are similar for algebra and geometry programs and somewhat lower for integrated programs. We can assess the normality of the distributions, and important assumption for hypothesis tests, by considering the skewness and kurtosis. Both should be about zero if the distribution is normal. The distributions for content deviate more seriously from normality than do the other factors. This can be seen more clearly in Figure 45.

Table 32. Score distribution characteristics for Algebra 1 and 2 series by Content/Standards Alignment and other factors.

	Content	Other factors
Mean (unweighted)	0.7457	0.6975
Standard deviation	0.2990	0.2848
Skewness	-0.9149	-0.7263
Kurtosis	-0.1640	-0.0867

Table 33. Score distribution characteristics for geometry programs by Content/Standards Alignment and other factors.

	Content	Other factors
Mean (unweighted)	0.756	0.732
Standard deviation	0.298	0.269
Skewness	-1.011	-0.830
Kurtosis	0.062	0.225

Table 34. Score distribution characteristics for integrated programs by Content/Standards Alignment and other factors.

	Content	Other factors
Mean (unweighted)	0.606	0.673
Standard deviation	0.346	0.282
Skewness	-0.225	-0.541
Kurtosis	-1.230	-0.361

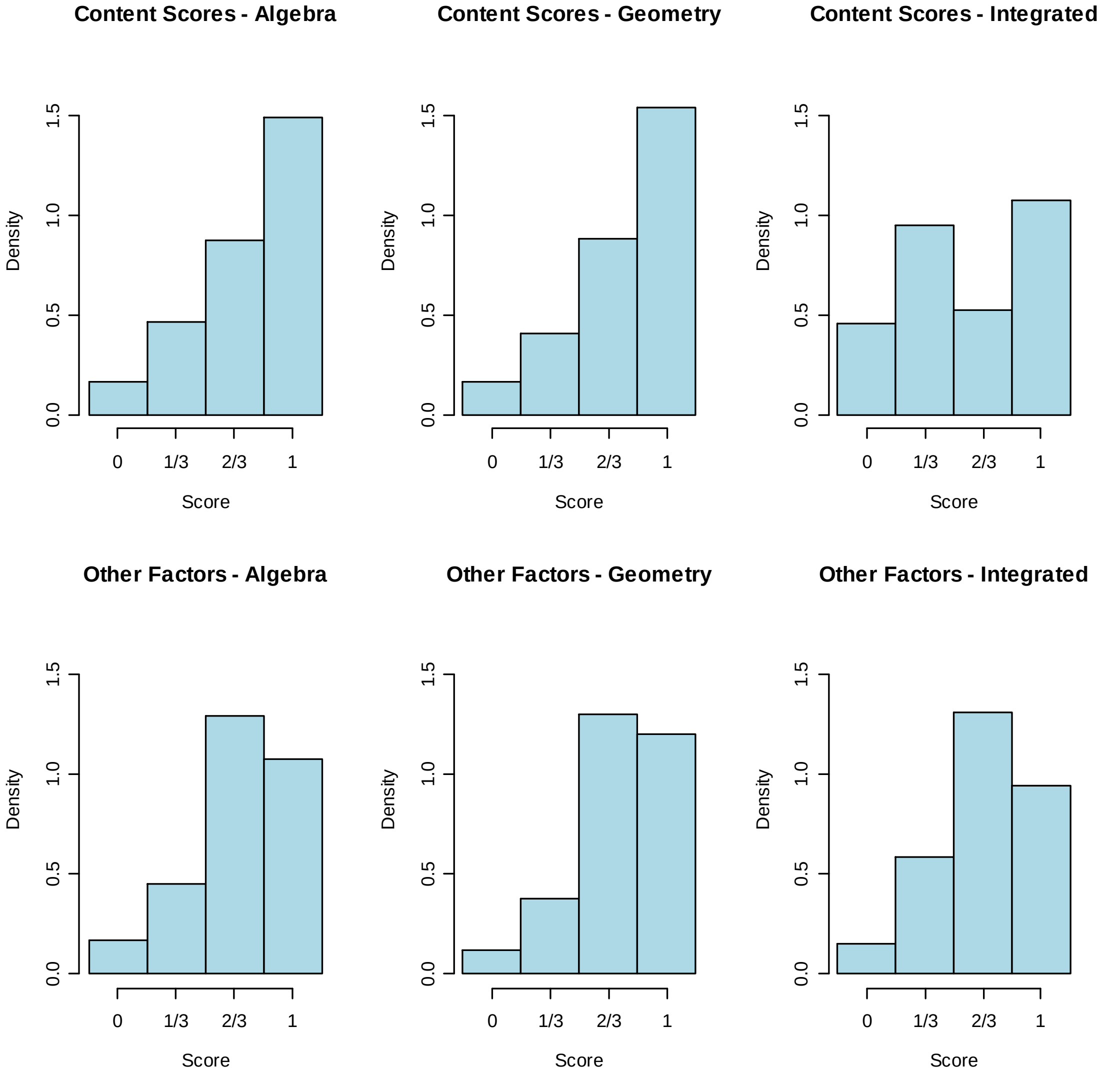

Figure 45. Histograms of adjusted scores on content and other factors scales by program type.

While the distributions are not normal, we will be comparing averages over hundreds of scores, which should make assumptions of normality not unreasonable.

5.4 Reviewer Bias

Table 35 gives the distribution of scores by reviewer on content items. There do not appear to be any reviewers who stand out in the distribution of scores assigned, with the exceptions of 998 and 999. These two reviewers reviewed only one text apiece, so this likely reflects variability in the texts rather than the raters.

Table 35. Distribution of scores by reviewer for content/standards alignment items.

	Raw score			
Reviewer	Not met	Limited content	Limited practice	Met
15	9.4%	12.9%	14.7%	62.9%
18	6.6%	5.7%	30.3%	57.5%
28	11.2%	14.5%	21.1%	53.3%
33	5.8%	15.0%	26.6%	52.6%
52	2.9%	12.9%	33.8%	50.5%
77	6.5%	26.1%	35.2%	32.1%
97	8.3%	9.1%	19.4%	63.2%
117	3.9%	14.6%	28.6%	52.9%
127	3.1%	11.2%	43.4%	42.4%
143	12.2%	12.5%	22.0%	53.3%
168	5.6%	12.8%	33.0%	48.6%
188	3.8%	5.0%	25.2%	66.0%
206	3.9%	14.4%	38.8%	42.8%
232	5.1%	11.0%	61.0%	22.8%
240	17.9%	18.5%	20.8%	42.9%
242	1.5%	10.4%	40.0%	48.1%
274	6.9%	19.0%	19.0%	55.0%
282	4.3%	23.6%	31.4%	40.7%
285	5.1%	9.8%	18.5%	66.5%
287	7.9%	11.3%	29.9%	50.9%
298	6.4%	15.5%	42.8%	35.4%
301	1.2%	12.1%	36.5%	50.2%
320	7.0%	11.1%	29.9%	52.0%
322	3.5%	8.9%	28.3%	59.3%
336	2.2%	12.4%	29.2%	56.2%
360	7.0%	9.9%	23.2%	59.9%
382	18.6%	20.6%	24.3%	36.4%
394	7.8%	13.3%	17.3%	61.6%
442	5.7%	11.1%	19.3%	63.9%
446	5.5%	14.5%	19.6%	60.4%
448	8.5%	24.6%	27.3%	39.6%

	Raw score			
Reviewer	Not met	Limited content	Limited practice	Met
449	5.8%	14.7%	34.4%	45.1%
450	2.1%	6.9%	36.2%	54.8%
452	8.4%	24.3%	35.1%	32.2%
457	3.0%	5.0%	17.8%	74.3%
458	4.7%	14.7%	34.9%	45.7%
998	43.9%	19.5%	26.8%	9.8%
999	0.0%	30.6%	19.4%	50.0%
Total	6.2%	13.7%	30.0%	50.2%

We can confirm visually that no single reviewer stands apart from the rest from Figure 46, which gives the average score on standards by reviewer with bands of one standard deviation indicating the variability for each reviewer. While there is one reviewer with a much lower average score than the others, the variability indicates that it is possible that this is simply due to chance. Moreover, this is a person who reviewed one text only, and the score given is consistent with the scores on that particular text given by other reviewers.

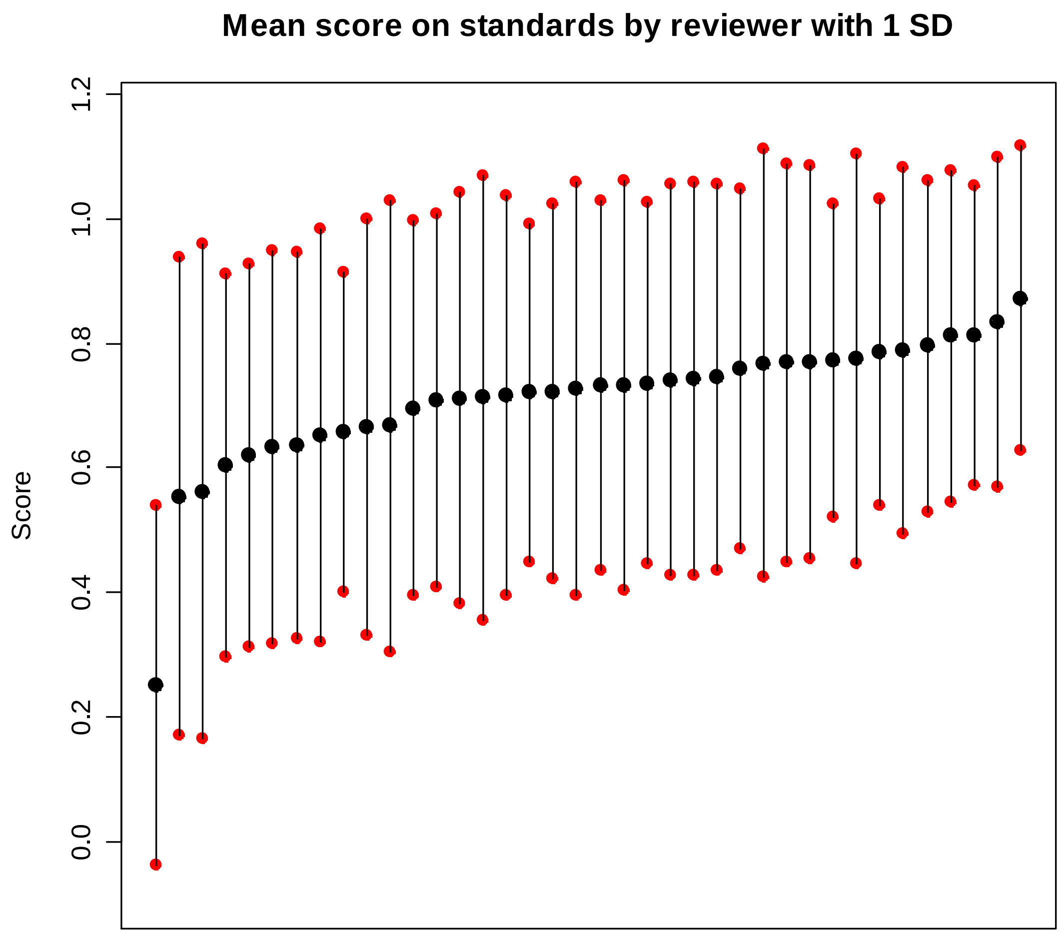

Figure 46. Average standards score by reviewer with bands of one standard deviation.

Table 36 gives the distribution of scores by reviewer on other factors items. The scores here are somewhat more variable, with several reviewers not using "strongly disagree" at all.

Table 36. Distribution of scores by reviewer for other factors items.

Reviewer	Raw score: Strongly disagree	Disagree	Agree	Strongly Agree
15	6.0%	22.6%	37.7%	33.7%
18	3.6%	6.7%	15.5%	74.2%
28	1.2%	9.5%	69.6%	19.6%
33	15.0%	16.0%	37.1%	32.0%
52	0.0%	11.4%	77.1%	11.4%
77	4.5%	19.0%	45.5%	31.0%

	Raw score			
Reviewer	**Strongly disagree**	**Disagree**	**Agree**	**Strongly Agree**
97	2.0%	20.2%	34.5%	43.3%
117	4.8%	14.3%	45.9%	35.0%
127	0.0%	11.6%	54.4%	34.0%
143	10.5%	12.9%	22.1%	54.4%
168	1.0%	29.9%	60.2%	8.8%
188	0.8%	27.4%	63.1%	8.7%
206	5.3%	13.5%	45.5%	35.7%
232	1.6%	17.2%	74.9%	6.3%
240	6.0%	8.9%	27.4%	57.7%
242	4.4%	10.5%	47.6%	37.4%
274	3.6%	22.6%	57.9%	15.9%
282	4.0%	20.6%	56.3%	19.0%
285	2.4%	10.3%	38.9%	48.4%
287	8.3%	15.2%	31.5%	44.9%
298	7.4%	31.3%	50.9%	10.3%
301	1.9%	6.1%	38.1%	53.9%
320	9.5%	6.5%	28.9%	55.1%
322	0.0%	4.0%	32.1%	63.9%
336	6.5%	15.3%	48.3%	29.9%
360	2.4%	25.5%	45.6%	26.5%
382	12.3%	24.2%	46.0%	17.5%
394	9.5%	17.5%	27.0%	46.0%
442	2.4%	6.8%	23.8%	67.0%
446	8.3%	8.3%	23.4%	59.9%
448	11.9%	21.4%	33.0%	33.7%
449	6.0%	6.8%	62.9%	24.3%
450	2.4%	7.6%	68.6%	21.4%
452	11.4%	20.0%	32.9%	35.7%
457	3.0%	8.6%	23.2%	65.2%
458	1.6%	16.7%	50.4%	31.3%
998	31.0%	23.8%	21.4%	23.8%
999	11.9%	28.6%	52.4%	7.1%
Total	5.2%	15.4%	43.4%	36.0%

Figure 47 shows the average score by reviewer on other factors, together with a one standard deviation band to indicate variability. In this case, no single reviewer stands out.

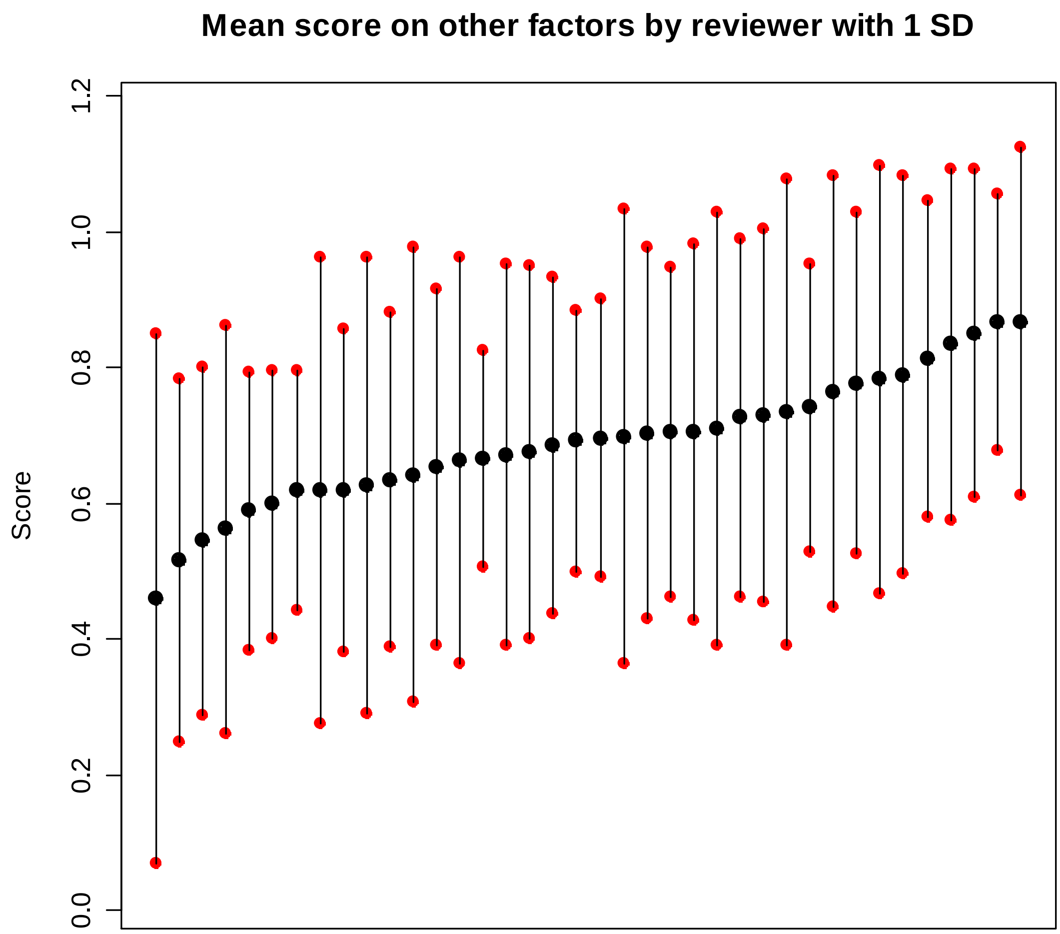

Figure 47. Average other factors score by reviewer with bands of one standard deviation.

In order to test whether any reviewer had a tendency to over- or under-rate, we calculated a standardized score within text for each reviewer, and performed a t-test comparing each average standardized score to 0 to test whether the reviewer tended to score away from the mean. This is only possible for reviewers who completed multiple reviews, so reviewers 998 and 999 are not shown. The results are shown in Table 37 and Table 38 for content and other factors, respectively. Since we are performing tests for the 36 reviewers with multiple reviews, it is important to adjust for multiple comparisons to avoid finding a difference significant when it could have happened by chance when drawing 36 means from the same distribution. The tables give the adjusted significance level, calculated using the Holm-Bonferroni method, in which we compare the ordered p-values to the nominal significance level (0.05) divided by the number of tests remaining. As soon as one test is deemed insignificant, the rest are also.

In this case, we see that even the smallest p-value for content reviews does not reach the adjusted significance level of 0.05/36, so we can conclude that there is no evidence of reviewer bias on content/standards alignment. The score given by reviewer 999 is safely in the middle of the scores for the text reviewed, indicating no significant bias, while the score given by reviewer 998 was the lowest for that text. It does not appear to be substantially lower than the rest, however.

Table 37. t-tests for evidence of reviewer bias on Content/Standards Alignment.

Reviewer	t-value	df	p-value	Tests remaining	Significance level
457	2.93	8	0.0095	36	0.0014
282	-3.04	6	0.0114	35	0.0014
188	2.52	6	0.0227	34	0.0015
448	-2.20	7	0.0319	33	0.0015
298	-1.84	9	0.0491	32	0.0016
168	1.78	7	0.0595	31	0.0016
232	-1.64	9	0.0674	30	0.0017
143	1.57	7	0.0802	29	0.0017
442	1.50	8	0.0866	28	0.0018
452	-1.48	5	0.0991	27	0.0019
360	1.39	7	0.1031	26	0.0019
242	-1.37	7	0.1068	25	0.0020
301	-1.31	11	0.1085	24	0.0021
15	-1.28	6	0.1245	23	0.0022
77	-1.22	9	0.1271	22	0.0023
285	1.15	6	0.1477	21	0.0024
28	-1.19	4	0.1495	20	0.0025
394	-1.13	6	0.1510	19	0.0026
458	-0.99	6	0.1792	18	0.0028
382	-0.97	6	0.1844	17	0.0029
117	-0.86	7	0.2091	16	0.0031
52	-0.75	5	0.2437	15	0.0033
287	-0.71	8	0.2503	14	0.0036
127	-0.58	7	0.2899	13	0.0038
450	-0.47	5	0.3279	12	0.0042
240	0.47	4	0.3301	11	0.0045
449	-0.43	6	0.3414	10	0.0050
446	0.42	6	0.3434	9	0.0056
320	-0.42	7	0.3441	8	0.0063
18	0.35	6	0.3698	7	0.0071
97	-0.28	6	0.3949	6	0.0083
322	0.25	6	0.4057	5	0.0100
33	0.24	7	0.4076	4	0.0125
206	-0.23	9	0.4106	3	0.0167
274	0.17	6	0.4352	2	0.0250
336	0.11	7	0.4559	1	0.0500

It appears, however, that there are two reviewers with a tendency to rate texts higher on other factors. In this case, the score given by reviewer 999 is again in the middle of the scores for the text reviewed and the score given by reviewer 998 was the lowest for that text. It does not appear to be substantially lower than the rest, however, indicating that neither reviewer is likely to have been significantly biased.

Table 38. t-tests for evidence of reviewer bias on other factors.

Reviewer	t-value	df	p-value	Tests remaining	Significance level
442	6.11	8	0.0001	36	0.0014
322	7.05	6	0.0002	35	0.0014
298	-3.12	9	0.0062	34	0.0015
232	-3.09	9	0.0064	33	0.0015
282	-3.11	6	0.0104	32	0.0016
143	2.94	7	0.0109	31	0.0016
336	-2.87	7	0.0120	30	0.0017
301	2.59	11	0.0125	29	0.0017
457	2.70	8	0.0135	28	0.0018
382	-2.62	6	0.0198	27	0.0019
168	-2.31	7	0.0273	26	0.0019
240	2.53	4	0.0323	25	0.0020
28	-2.53	4	0.0324	24	0.0021
188	-2.25	6	0.0328	23	0.0022
18	2.09	6	0.0410	22	0.0023
15	-1.71	6	0.0694	21	0.0024
446	1.52	6	0.0897	20	0.0025
320	1.43	7	0.0976	19	0.0026
394	-1.38	6	0.1080	18	0.0028
285	1.28	6	0.1237	17	0.0029
450	-1.11	5	0.1579	16	0.0031
52	-1.04	5	0.1732	15	0.0033
127	1.01	7	0.1733	14	0.0036
33	-0.81	7	0.2235	13	0.0038
274	-0.81	6	0.2257	12	0.0042
448	-0.77	7	0.2340	11	0.0045
206	0.72	9	0.2436	10	0.0050
117	-0.71	7	0.2511	9	0.0056
452	-0.62	5	0.2825	8	0.0063
449	-0.46	6	0.3293	7	0.0071
77	0.43	9	0.3377	6	0.0083
287	0.37	8	0.3607	5	0.0100
360	-0.32	7	0.3796	4	0.0125
458	0.30	6	0.3858	3	0.0167
97	0.11	6	0.4589	2	0.0250
242	0.00	7	0.4983	1	0.0500

5.5 Content/Standards Alignment

The first step in our analysis is to evaluate the agreement of each program with the state math standards. The following tables give the average score on Content/Standards Alignment items for algebra, geometry and integrated programs, respectively, along with the 95% normal confidence interval for the mean.

Table 39. Summary of Content/Standards Alignment scores for Algebra 1 and 2 series.

					95% CI	
Program	**Mean**	**Std. dev**	**N**	**Std. err.**	**Lower**	**Upper**
Discovering - Algebra	0.863	0.238	416	0.012	0.840	0.886
Holt Algebra	0.841	0.239	416	0.012	0.818	0.864
PH Math Algebra	0.833	0.238	416	0.012	0.810	0.856
Glencoe McGraw-Hill Algebra	0.823	0.228	420	0.011	0.802	0.845
McDougal Littell Algebra	0.786	0.270	420	0.013	0.760	0.811
CPM Algebra	0.751	0.329	416	0.016	0.719	0.782
CME Algebra	0.739	0.308	420	0.015	0.710	0.769
Cognitive Tutor Algebra	0.735	0.254	416	0.012	0.711	0.760
PH Classics (Foerster) Algebra	0.709	0.330	456	0.015	0.678	0.739
CORD Algebra	0.705	0.293	380	0.015	0.675	0.734
PH Classics (Smith) Algebra	0.692	0.316	532	0.014	0.665	0.719
MathConnections Algebra	0.528	0.328	496	0.015	0.499	0.556

Table 40. Summary of Content/Standards Alignment scores for geometry programs.

					95% CI	
Program	**Mean**	**Std. dev**	**N**	**Std. err.**	**Lower**	**Upper**
Holt Geometry	0.860	0.198	258	0.012	0.836	0.885
PH Math Geometry	0.854	0.238	215	0.016	0.822	0.886
McDougal Littell Geometry	0.850	0.247	215	0.017	0.817	0.883
Glencoe McGraw-Hill Geometry	0.847	0.211	301	0.012	0.823	0.871
CORD Geometry	0.810	0.291	258	0.018	0.775	0.846
Discovering - Geometry	0.783	0.282	215	0.019	0.745	0.821
CPM Geometry	0.744	0.295	301	0.017	0.711	0.778
Cognitive Tutor Geometry	0.699	0.338	258	0.021	0.658	0.740
CME Geometry	0.625	0.310	258	0.019	0.588	0.663
MathConnections Geometry	0.512	0.318	258	0.020	0.473	0.550

Table 41. Summary of Content/Standards Alignment scores for integrated programs.

					95% CI	
Program	**Mean**	**Std. dev**	**N**	**Std. err.**	**Lower**	**Upper**
Core Plus Math	0.671	0.319	667	0.012	0.646	0.695
SIMMS Math	0.656	0.330	667	0.013	0.631	0.681
Interactive Math Program	0.490	0.359	667	0.014	0.463	0.518

An eligibility criterion of an average score of at least 0.7 on content was originally proposed. We use one-sided t-tests to compare each program's average score to the threshold value of 0.7; the results are given in Tables 11 through 13. Of the Algebra 1 and 2 series, only Math Connections Algebra has a mean that is significantly lower than 0.7, while both Math Connections Geometry and CME Geometry do not meet the cutoff. All three integrated programs are significantly below the threshold value.

5.6 Threshold Tests

The tables below give the results of t-tests comparing the average Content/Standards Alignment scores for algebra, geometry and integrated math, respectively, to the threshold value of 0.7.

Only one Algebra 1 and 2 series, Math Connections, has a score for content that is significantly below the threshold. Both Math Connections and CME fail to meet the threshold on Geometry programs, while all three Integrated programs do not meet the threshold when treated as individual courses (reductions in scores are applied when the standard is found above or below the expected course level. However, when the integrated programs are treated as a whole series (full score is given regardless of where the standard was met in the series), only Integrated Math Program fails to exceed the content threshold.

Table 42. Summary of Content/Standards Alignment scores for Algebra 1 and 2 programs.

Program	Mean	Std err.	t-value	Degrees of Freedom	p-value	Tests remaining	Significance level
Math Connections Algebra	0.528	0.015	-11.71	495	2.08E-28	12	0.004
PH Classics (Smith) Algebra	0.692	0.014	-0.60	531	0.273	11	0.005
CORD Algebra	0.705	0.015	0.32	379	0.626	10	0.005
PH Classics (Foerster) Algebra	0.709	0.015	0.56	455	0.713	9	0.006
CME Algebra	0.739	0.015	2.61	419	0.995	8	0.006
Cognitive Tutor Algebra	0.735	0.012	2.83	415	0.998	7	0.007
CPM Algebra	0.751	0.016	3.15	415	0.999	6	0.008
McDougal Littell Algebra	0.786	0.013	6.52	419	1.000	5	0.010
Discovering - Algebra	0.863	0.012	14.00	415	1.000	4	0.013
Holt Algebra	0.841	0.012	12.05	415	1.000	3	0.017
PH Math Algebra	0.833	0.012	11.43	415	1.000	2	0.025
Glencoe McGraw-Hill Algebra	0.823	0.011	11.11	419	1.000	1	0.050

Table 43. Summary of Content/Standards Alignment scores for geometry programs.

Program	Mean	Std err.	t-value	Degrees of Freedom	p-value	Tests remaining	Significance level
Math Connections Geometry	0.512	0.020	-9.51	257	7.24E-19	10	0.005
CME Geometry	0.625	0.019	-3.87	214	7.18E-05	9	0.006
Cognitive Tutor Geometry	0.699	0.021	-0.05	214	0.480	8	0.006
CPM Geometry	0.744	0.017	2.59	300	1.00	7	0.007

Program	Mean	Std err.	t-value	Degrees of Freedom	p-value	Tests remaining	Significance level
Discovering - Geometry	0.783	0.019	4.32	257	1.00	6	0.008
CORD Geometry	0.810	0.018	6.09	214	1.00	5	0.010
McDougal Littell Geometry	0.850	0.017	8.89	300	1.00	4	0.013
Holt Geometry	0.860	0.012	13.01	257	1.00	3	0.017
PH Math Geometry	0.854	0.016	9.51	257	1.00	2	0.025
Glencoe McGraw-Hill Geometry	0.847	0.012	12.08	257	1.00	1	0.050

Table 44. Summary of Content/Standards Alignment scores for integrated programs, treated as individual courses (score reductions applied when standard was found above/below expected level).

Program	Mean	Std err.	t-value	Degrees of Freedom	p-value	Tests remaining	Significance level
Interactive Math Program	0.490	0.014	-15.07	666	1.09E-44	3	0.017
SIMMS Math	0.656	0.013	-3.43	666	3.16E-04	2	0.025
Core Plus Math	0.671	0.012	-2.38	666	8.89E-03	1	0.050

Table 45. Summary of Content/Standards Alignment scores for integrated programs, treated as a series (no score reductions applied when standard was found above/below expected level).

Program	Mean	Std err.	t-value	Degrees of Freedom	p-value	Tests remaining	Significance level
Interactive Math Program	0.609	0.014	-6.45527	666	1.04E-10	3	0.017
SIMMS Math	0.710	0.012	0.818857	666	0.79	2	0.025
Core Plus Math	0.802	0.011	9.133529	666	1.00	1	0.050

5.7 Calculation of Program Means and Standard Errors

For the comparison of programs, we consider the weighted averages of scores across all scales and their standard errors. The six scales are weighted as shown in Table 46. The average score for each program is calculated as the weighted sum of the average scores in the six scales.

Table 46. Scale weights for overall averages.

Scale	Weight
Assessment	0.050
Content/Standards Alignment	0.700
Equity and Access	0.040
Instructional Planning and Professional Support	0.045
Program Organization and Design	0.090
Student Learning	0.075

To calculate the standard error of the average score for each program, we first take the variance of the average score for each scale. The variance for the program is the sum of the square of the weight for the scale from Table 46 times the variance of the scale. The standard error is then the square root of this value.

The following tables give the calculated means and standard errors for algebra, geometry and integrated programs, respectively. Also included is a 95% confidence interval for the value of the mean.

Table 47. Summary of overall weighted mean scores for Algebra 1 and 2 series.

Program	Mean	Std. err.	95% CI Lower	95% CI Upper
Discovering - Algebra	0.859	0.009	0.842	0.876
Holt Algebra	0.832	0.009	0.815	0.849
Glencoe McGraw-Hill Algebra	0.821	0.008	0.804	0.837
PH Math Algebra	0.814	0.009	0.796	0.831
CPM Algebra	0.768	0.012	0.745	0.791
McDougal Littell Algebra	0.752	0.010	0.732	0.771
CME Algebra	0.731	0.011	0.710	0.753
Cognitive Tutor Algebra	0.714	0.009	0.696	0.733
CORD Algebra	0.699	0.011	0.677	0.721
PH Classics (Foerster) Algebra	0.672	0.011	0.650	0.695
PH Classics (Smith) Algebra	0.658	0.010	0.638	0.679
MathConnections Algebra	0.532	0.011	0.511	0.553

Table 48. Summary of overall weighted mean scores for geometry programs.

Program	Mean	Std. err.	95% CI Lower	95% CI Upper
Holt Geometry	0.847	0.010	0.828	0.866
McDougal Littell Geometry	0.843	0.013	0.818	0.868
Glencoe McGraw-Hill Geometry	0.832	0.009	0.813	0.850
PH Math Geometry	0.827	0.012	0.803	0.851
CORD Geometry	0.795	0.014	0.769	0.822
Discovering - Geometry	0.776	0.014	0.748	0.804
Cognitive Tutor Geometry	0.730	0.015	0.700	0.761
CPM Geometry	0.729	0.013	0.704	0.755
CME Geometry	0.613	0.014	0.586	0.641
MathConnections Geometry	0.528	0.015	0.499	0.557

Table 49. Summary of overall weighted mean scores for integrated programs.

Program	Mean	Std. err.	95% CI Lower	95% CI Upper
Core Plus Math	0.688	0.009	0.670	0.706
SIMMS Math	0.658	0.009	0.639	0.676
Interactive Math Program	0.538	0.010	0.518	0.558

5.8 Program Comparison

Since the goal is to identify no more than three program recommendations, we need to test for any statistical ties for third place. To do this, we compare the scores of the lower-ranked programs to the third-ranked (as determined by the weighted average score across scales). We perform the comparisons using t-tests, adjusting for multiple comparisons using the Holm-Bonferroni method. To do so, we compare the ordered p-values to the nominal significance level (0.05) divided by the number of tests remaining. As soon as one test is deemed insignificant, the rest are as well.

The Welch-Sattherwaite equation gives us an approximation to the degrees of freedom for a t-test comparing weighted averages.

Take s_1 and $s_2$to be the standard errors of the two programs to be compared.

The degrees of freedom are then given by

$$\frac{(s_1^2 + s_2^2)^2}{m_1 + m_2}$$

where

$$m_k = \sum_i \frac{{w_i}^2 * s_i^2}{n_i}$$

The index i ranges over the six response scales. w_i is the category weight, $n_i$is the number of scores in that category and s_i is the standard deviation of observations in that category.

The results for algebra and geometry programs are given in the following tables. In both cases, there is one program, PH Math, which is tied with the top three programs. Since there are only three integrated programs, there is no need to do any tests for ties. We do, however, give the weighted mean scores in Table 52.

Table 50. t-test results comparing lower-scoring programs to the third-highest scoring Algebra 1 and 2 series.

	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Discovering - Algebra	0.859					
Holt Algebra	0.832					
Glencoe McGraw-Hill Algebra	0.821					
MathConnections Algebra	0.532	-21.08	98	2.69E-38	9	0.006
PH Classics (Smith) Algebra	0.658	-12.28	90	3.11E-21	8	0.006
PH Classics (Foerster) Algebra	0.672	-10.48	93	1.14E-17	7	0.007
CORD Algebra	0.699	-8.71	88	8.88E-14	6	0.008

	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Cognitive Tutor Algebra	0.714	-8.47	89	2.49E-13	5	0.010
CME Algebra	0.731	-6.47	95	2.10E-09	4	0.013
McDougal Littell Algebra	0.752	-5.31	89	4.05E-07	3	0.017
CPM Algebra	0.768	-3.63	94	2.31E-04	2	0.025
PH Math Algebra	0.814	-0.59	86	0.277	1	0.050

Table 51. t-test results comparing lower-scoring programs to the third-highest scoring geometry program.

	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Holt Geometry	0.847					
McDougal Littell Geometry	0.843					
Glencoe McGraw-Hill Geometry	0.832					
MathConnections Geometry	0.528	-17.33	73	1.07E-27	7	0.007
CME Geometry	0.613	-12.79	76	7.78E-21	6	0.008
CPM Geometry	0.729	-6.41	83	4.35E-09	5	0.010
Cognitive Tutor Geometry	0.730	-5.61	70	1.95E-07	4	0.013
Discovering - Geometry	0.776	-3.25	76	8.63E-04	3	0.017
CORD Geometry	0.795	-2.21	80	0.015	2	0.025
PH Math Geometry	0.827	-0.31	87	0.377	1	0.050

Table 52. Weighted mean scores for integrated programs when treated as individual courses.

Program name	Mean score
Core Plus Math	0.688
SIMMS Math	0.658
Interactive Math Program	0.538

Recall that we found two reviewers, 442 and 322, to be biased in their scoring of other factors. Both tended to rate texts more highly than the other reviewers rating those texts. However, reviewer 442 rated at least 2 of the 3 texts in all integrated programs, plus one algebra program. Thus, the bias is fairly evenly spread over the integrated programs, and is not likely to significantly impact the results. Reviewer 322 rated 6 of 10 geometry programs; 4 of them are significantly lower-scoring than the top 3, so the bias cannot have given them a falsely high ranking. The other two fall in the top 3, and hence must be checked for inflated position due to biased scoring. Remember, however, that other factors account for only 30% of the final score, so the impact is likely to be minimal.

We repeat the program comparison with other factors ratings from reviewers 442 and 322 removed; the results are given in Tables 21 through 23. The results for algebra programs are virtually unchanged, since only one review of one text is affected. The weighted mean scores for the integrated programs have decreased somewhat, but the order remains unchanged, as we would expect from the equitable distribution of inflation from reviewer

442. The substantive results for geometry programs remain the same, though the mean scores decline somewhat.

Table 53. t-test results comparing lower-scoring programs to the third-highest scoring Algebra 1 and 2 series after removing reviewers 442 and 322.

	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Discovering - Algebra	0.859					
Holt Algebra	0.832					
Glencoe McGraw-Hill Algebra	0.821					
MathConnections Algebra	0.532	-21.08	98	2.69E-38	9	0.006
PH Classics (Smith) Algebra	0.658	-12.28	90	3.11E-21	8	0.006
PH Classics (Foerster) Algebra	0.672	-10.48	93	1.14E-17	7	0.007
CORD Algebra	0.699	-8.71	88	8.88E-14	6	0.008
Cognitive Tutor Algebra	0.714	-8.47	89	2.49E-13	5	0.010
CME Algebra	0.731	-6.47	95	2.10E-09	4	0.013
McDougal Littell Algebra	0.752	-5.31	89	4.05E-07	3	0.017
CPM Algebra	0.765	-3.84	94	1.11E-04	2	0.025
PH Math Algebra	0.814	-0.59	86	0.277	1	0.050

Table 54. t-test results comparing lower-scoring programs to the third-highest scoring geometry program after removing reviewers 442 and 322.

	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Holt Geometry	0.846					
McDougal Littell Geometry	0.841					
Glencoe McGraw-Hill Geometry	0.829					
MathConnections Geometry	0.528	-17.05	73	2.71E-27	7	0.007
CME Geometry	0.613	-12.53	75	2.21E-20	6	0.008
CPM Geometry	0.725	-6.35	81	5.76E-09	5	0.010
Cognitive Tutor Geometry	0.724	-5.74	69	1.20E-07	4	0.013
Discovering - Geometry	0.761	-3.93	75	9.33E-05	3	0.017
CORD Geometry	0.795	-2.01	80	0.024	2	0.025
PH Math Geometry	0.827	-0.12	87	0.454	1	0.050

Table 55. Weighted mean scores for integrated programs after removing reviewers 442 and 322.

Program name	Mean score
Core Plus Math	0.679
SIMMS Math	0.647
Interactive Math Program	0.532

5.9 *Standard Error Calculations*

This section describes several methodological variants to calculate standard error. The recommended approach is the most straightforward. The more complex variants take into account assumptions about dependence in the data, but ultimately show that substantive results are unaffected for algebra and integrated programs, while one additional geometry program, CORD, is found to be tied with the third-ranked text under certain situations.

5.9.1 Recommended Approach

5.9.1.1 Methodology

Let $X_{ijkl}^{(p)}$ be the score for program p on item l for scale i, grade j, by rater k.

Here:

- p indexes the 25 curricula
- i = 1,…, 6, indexes the 6 scales assessed (Content/Standards Alignment, Equity and Access, etc.)
- j = 1,…,J, indexes the grade levels.
- J=1, 2 or 3 for geometry, algebra or integrated programs, respectively.
- k = 1,…,K_j. K_j indexes the reviewers, and ranges from 5 to 7 depending on the text and grade level.
- l = 1,…,L_{ij}. L_{ij} index the number of items scored, and varies depending upon the grade level and scale.

The final weighted average score for program p is

$$\overline{X}_w^{(p)} = \sum_{i=1}^{6} w_i \overline{X}_{i...}$$

where w_i is the weight given to scale i, and $\overline{X}_{i...}$ is the average rating given on items in scale i on program p, averaged over grade levels and raters.

More formally,

$$\overline{X}_w^{(p)} = \sum_{i=1}^{6} w_i \sum_{j=1}^{J} \sum_{k=1}^{K_j} \sum_{l=1}^{L_{ij}} X_{ijkl} / N_i ,$$

where

$$N_i = \sum_{j=1}^{J} K_j L_{ij}$$

is the number of item scores on scale i for program p.

5.9.1.2 Variance and standard error of weighted average for final score

The precision with which the final score for program p can be assessed depends upon the number of ratings and the variability of the ratings. More ratings correspond to higher precision (lower variance and standard error). Lower variability of ratings, indicating greater agreement among ratings, corresponds to higher precision. In addition, the weights given to the 6 different categories impact the variance and standard error. Note also that the standard error (SE) is the square root of the variance of the average.

For the current problem, the variance for the weighted average $\overline{X}_w^{(p)}$ (Final Score for program p) can be computed as follows.

$$Var(\overline{X}_w^{(p)}) = \sum_{i=1}^{6} w_i^2 Var(\overline{X}_{i...})$$

Three assumptions are inherent in this computation: (1) independence of the ratings $X_{ijkl}^{(p)}$ (2) independence of scales, and (3) all items within a scale are assessing program p on category i (in other words, all items are independent and identically distributed measures of a true scale average for program p).

$$Var(\overline{X}_{i...}) = \sigma_i^2 / N_i .$$

The usual estimator for σ_i^2 is the sample variance s_i^2, computed from the N_i scores $X_{ijkl}^{(p)}$

Thus the estimated standard error (SE) for $\overline{X}_w^{(p)}$, the Final Score for program p is

$$\sqrt{\sum_{i=1}^{6} w_i^2 s_i^2 / N_i}$$

5.9.1.3 Results

Table 56 and Table 57 give the t-test results, comparing all lower-rated programs to the third-rated program, again by program type. For both algebra and geometry, only the 4th rated program, PH Math, cannot statistically be distinguished from the third-rated program.

Table 56. t-test results comparing lower-scoring programs to the third-highest scoring Algebra 1 and 2 series.

Program	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Discovering - Algebra	0.859					
Holt Algebra	0.832					
Glencoe McGraw-Hill Algebra	0.821					

Program	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
MathConnections Algebra	0.532	-21.08	98	2.69E-38	9	0.006
PH Classics (Smith) Algebra	0.658	-12.28	90	3.11E-21	8	0.006
PH Classics (Foerster) Algebra	0.672	-10.48	93	1.14E-17	7	0.007
CORD Algebra	0.699	-8.71	88	8.88E-14	6	0.008
Cognitive Tutor Algebra	0.714	-8.47	89	2.49E-13	5	0.010
CME Algebra	0.731	-6.47	95	2.10E-09	4	0.013
McDougal Littell Algebra	0.752	-5.31	89	4.05E-07	3	0.017
CPM Algebra	0.768	-3.63	94	2.31E-04	2	0.025
PH Math Algebra	0.814	-0.59	86	0.277	1	0.050

Table 57 t-test results comparing lower-scoring programs to the third-highest scoring geometry program.

Program	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Holt Geometry	0.847					
McDougal Littell Geometry	0.843					
Glencoe McGraw-Hill Geometry	0.832					
MathConnections Geometry	0.528	-17.33	73	1.07E-27	7	0.007
CME Geometry	0.613	-12.79	76	7.78E-21	6	0.008
CPM Geometry	0.729	-6.41	83	4.35E-09	5	0.010
Cognitive Tutor Geometry	0.730	-5.61	70	1.95E-07	4	0.013
Discovering - Geometry	0.776	-3.25	76	8.63E-04	3	0.017
CORD Geometry	0.795	-2.21	80	0.015	2	0.025
PH Math Geometry	0.827	-0.31	87	0.377	1	0.050

5.9.2 Independence of Scales

5.9.2.1 Motivation

We might expect that a program that scores well on one scale would also score well on another scale, simply because it is a high-quality program. This would indicate that program scores on the six scales are not independent. In Table 58 we see the correlations between the six scales. With correlations ranging from 0.42 to 0.86, it is unlikely that the scales are independent.

Table 58. Scale correlations.

	Assessment	Content	Equity and Access	Planning and Support	Program Organization	Student Experience
Assessment	1.00	0.61	0.67	0.47	0.50	0.58
Content	0.61	1.00	0.62	0.42	0.48	0.53

	Assessment	Content	Equity and Access	Planning and Support	Program Organization	Student Experience
Equity	0.67	0.62	1.00	0.53	0.51	0.52
Planning	0.47	0.42	0.53	1.00	0.82	0.79
Program	0.50	0.48	0.51	0.82	1.00	0.86
Student	0.58	0.53	0.52	0.79	0.86	1.00

5.9.2.2 Methodology

The assumption of independence of the scales is what allows us to say that

$$Var(\overline{X}_w^{(p)}) = \sum_{i=1}^{6} w_i^2 Var(\overline{X}_{i...})$$

Without that assumption, we should adjust the variance for the covariances of the scales by taking:

$$Var(\overline{X}_{i...}) = \sum_{i=1}^{6}\sum_{m=1}^{6} w_i w_m Cov(\overline{X}_{i...}, \overline{X}_{m...})$$

Note that

$$Cov(\overline{X}_{i...}, \overline{X}_{i...}) = Var(\overline{X}_{i...})$$

5.9.2.3 Results

The following tables give the confidence interval and t-test results using this modified standard error calculation. We see that the results remain the same as above, except that now the 5th ranked geometry program, CORD Geometry is not significantly different from the third-ranked program.

Table 59. t-test results comparing lower-scoring programs to the third-highest scoring Algebra 1 and 2 series.

Program Name	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Discovering – Algebra	0.859					
Holt Algebra	0.832					
Glencoe McGraw-Hill Algebra	0.821					
MathConnections Algebra	0.532	-15.97	75	5.59E-26	9	0.006
PH Classics (Smith) Algebra	0.658	-9.16	69	7.62E-14	8	0.006
PH Classics (Foerster) Algebra	0.672	-7.93	70	1.23E-11	7	0.007
CORD Algebra	0.699	-6.47	67	6.68E-09	6	0.008
Cognitive Tutor Algebra	0.714	-6.23	67	1.85E-08	5	0.010
CME Algebra	0.731	-4.88	72	3.07E-06	4	0.013
McDougal Littell Algebra	0.752	-3.94	68	9.81E-05	3	0.017
CPM Algebra	0.768	-2.77	71	3.61E-03	2	0.025

Program Name	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
PH Math Algebra	0.814	-0.44	66	0.331	1	0.050

Table 60. t-test results comparing lower-scoring programs to the third-highest scoring geometry program.

Program Name	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Holt Geometry	0.847					
McDougal Littell Geometry	0.843					
Glencoe McGraw-Hill Geometry	0.832					
MathConnections Geometry	0.528	-12.72	54	4.83E-18	7	0.007
CME Geometry	0.613	-9.58	58	8.99E-14	6	0.008
CPM Geometry	0.729	-4.61	60	1.12E-05	5	0.010
Cognitive Tutor Geometry	0.730	-4.23	54	4.60E-05	4	0.013
Discovering - Geometry	0.776	-2.43	58	9.08E-03	3	0.017
CORD Geometry	0.795	-1.61	59	0.056	2	0.025
PH Math Geometry	0.827	-0.23	64	0.410	1	0.050

5.9.3 Identical Mean Distributions

5.9.3.1 Motivation

Since each item is a measure of a different aspect of alignment with a particular scale (i.e. different math standards in Content/Standards Alignment), it would be reasonable to assume that each item has a different mean value that contributes to the overall mean, rather than considering them all to be independent draws from one distribution.

5.9.3.2 Methodology

In this situation, rather than consider only the variance of the mean within scale, we begin with the variance of the scores themselves.

$$Var(\bar{X}_w^{(p)}) = \sum_{i=1}^{6} w_i \sum_{j=1}^{J} \sum_{k=1}^{K_j} \sum_{l=1}^{L_{ij}} Var(X_{ijkl} / N_i)$$

We estimate

$$Var(X_{ijkl} / N_i) = \sigma_{il}^2 / N_i^2$$

by

$$s_{il}^2 / N_i^2 ,$$

where s_{il}^2 is the sample variance of all scores on item l of category i (across programs).

5.9.3.3 Results

The following tables give results based on this standard error calculation. We see that the results are identical to the simplest standard error calculation given in *Section 5.9.1*.

Table 61. t-test results comparing lower-scoring programs to the third-highest scoring Algebra 1 and 2 series.

Program Name	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Discovering - Algebra	0.859					
Holt Algebra	0.832					
Glencoe McGraw-Hill Algebra	0.821					
MathConnections Algebra	0.532	-21.49	103	5.88E-40	9	0.006
PH Classics (Smith) Algebra	0.658	-12.28	104	2.90E-22	8	0.006
PH Classics (Foerster) Algebra	0.672	-10.84	101	6.33E-19	7	0.007
CORD Algebra	0.699	-8.48	97	1.32E-13	6	0.008
Cognitive Tutor Algebra	0.714	-7.60	99	9.10E-12	5	0.010
CME Algebra	0.731	-6.41	99	2.50E-09	4	0.013
McDougal Littell Algebra	0.752	-4.93	99	1.64E-06	3	0.017
CPM Algebra	0.768	-3.75	99	1.52E-04	2	0.025
PH Math Algebra	0.814	-0.52	99	0.303	1	0.050

Table 62. t-test results comparing lower-scoring programs to the third-highest scoring geometry program.

Program Name	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Holt Geometry	0.847					
McDougal Littell Geometry	0.843					
Glencoe McGraw-Hill Geometry	0.832					
MathConnections Geometry	0.528	-17.62	80	1.43E-29	7	0.007
CME Geometry	0.613	-12.66	80	4.27E-21	6	0.008
CPM Geometry	0.729	-6.18	84	1.13E-08	5	0.010
Cognitive Tutor Geometry	0.730	-5.88	80	4.62E-08	4	0.013
Discovering - Geometry	0.776	-3.05	75	1.56E-03	3	0.017
CORD Geometry	0.795	-2.11	80	0.019	2	0.025
PH Math Geometry	0.827	-0.27	75	0.395	1	0.050

5.9.4 Scale Independence and Identical Distributions

5.9.4.1 Motivation

We might expect that both of the previously discussed assumptions are violated and that the combined adjustment could change the results.

5.9.4.2 Methodology

The assumption of independence of the scales is what allows us to say that

$$Var(\overline{X}_w^{(p)}) = \sum_{i=1}^{6} w_i^2 Var(\overline{X}_{i...})$$

Without that assumption, we should adjust the variance for the covariances of the scales by taking:

$$Var(\overline{X}_{i...}) = \sum_{i=1}^{6}\sum_{m=1}^{6} w_i w_m Cov(\overline{X}_{i...}, \overline{X}_{m...})$$

Note that

$$Cov(\overline{X}_{i...}, \overline{X}_{i...}) = Var(\overline{X}_{i...}).$$

In this situation, rather than consider only the variance of the mean within scale, we begin with the variance of the scores themselves, to obtain:

$$Var(\overline{X}_{i...}) = \sum_{j=1}^{J}\sum_{k=1}^{K_j}\sum_{l=1}^{L_{ij}} Var(X_{ijkl} / N_i)$$

We estimate

$$Var(X_{ijkl} / N_i) = \sigma_{il}^{\ 2} / N_i^2$$

by

$$s_{il}^{\ 2} / N_i^2$$

where s_{il}^2 is the sample variance of all scores on item l of category i (across programs).

We can use

$$Var(\overline{X}_{i...})$$

to calculate the covariance, because

$$Cov(\overline{X}_{i...}, \overline{X}_{i...}) = \rho\sqrt{Var(\overline{X}_{i...})Var(\overline{X}_{j...})}$$

where ρ is the correlation between scales i and j.

5.9.4.3 Results

The following tables give the confidence intervals and t-test results. The conclusions are identical to those in *Section* ***Error! Reference source not found.***.

Table 63. t-test results comparing lower-scoring programs to the third-highest scoring Algebra 1 and 2 series.

Program Name	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
Discovering - Algebra	0.859					
Holt Algebra	0.832					
Glencoe McGraw-Hill Algebra	0.821					

Program Name	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
MathConnections Algebra	0.532	-11.94	57	1.94E-17	9	0.006
PH Classics (Smith) Algebra	0.658	-6.75	58	3.79E-09	8	0.006
PH Classics (Foerster) Algebra	0.672	-6.70	58	4.70E-09	7	0.007
CORD Algebra	0.699	-4.92	62	3.36E-06	6	0.008
CME Algebra	0.731	-3.65	57	2.86E-04	5	0.010
Cognitive Tutor Algebra	0.714	-3.55	57	3.90E-04	4	0.013
McDougal Littell Algebra	0.752	-3.11	44	1.67E-03	3	0.017
CPM Algebra	0.761	-1.93	62	0.029	2	0.025
PH Math Algebra	0.814	-0.30	42	0.385	1	0.050

Table 64. t-test results comparing lower-scoring programs to the third-highest scoring geometry program.

Program Name	Mean score	t statistic	Degrees of freedom	p-value	# tests remaining	Significance cutoff
McDougal Littell Geometry	0.861					
Holt Geometry	0.844					
PH Math Geometry	0.827					
MathConnections Geometry	0.528	-10.71	48	1.69E-14	7	0.007
CME Geometry	0.613	-7.67	48	4.03E-10	6	0.008
Cognitive Tutor Geometry	0.718	-3.65	46	3.35E-04	5	0.010
CPM Geometry	0.724	-3.53	47	4.72E-04	4	0.013
Discovering - Geometry	0.740	-3.04	47	1.94E-03	3	0.017
CORD Geometry	0.795	-1.06	46	0.147	2	0.025
Glencoe McGraw-Hill Geometry	0.826	-0.02	45	0.492	1	0.050

Appendix A. Programs Reviewed

Table 65. List of core/comprehensive materials submitted for review, including publisher information.

Program Name	Publisher Name	Copyright Date	Course and/or Course Series to be Reviewed	Type of Program	Contact Name	Email	Phone Number
CME Project	Pearson Prentice Hall	2009	Algebra 1 and 2	Text Based	Dorothy Kulwin or Kyle Bender	dorothy.kulwin@pearson.com kyle.bender@pearson.com	206/819-6814 or 253/906-1059
CME Project	Pearson Prentice Hall	2009	Geometry	Text Based	Dorothy Kulwin or Kyle Bender	dorothy.kulwin@pearson.com kyle.bender@pearson.com	206/819-6814 or 253/906-1059
Cognitive Tutor	Carnegie Learning, Inc.	2008	Algebra 1 and 2	Text AND Computer Components	Scott Wallace	swallace@carnegielearning.com	360/260-0435
Cognitive Tutor	Carnegie Learning, Inc.	2008	Geometry	Text AND Computer Components	Scott Wallace	swallace@carnegielearning.com	360/260-0435
CORD Algebra 1 and 2	CORD Communications, Inc.	Algebra 1: 2009 Algebra 2: 2008	Algebra 1 and 2	Text Based	Claudia Maness	cdmaness@cordcommunications.com	254/776-1822 ext. 371
CORD Geometry	CORD Communications, Inc.	2009	Geometry	Text Based	Claudia Maness	cdmaness@cordcommunications.com	254/776-1822 ext. 371
Core Plus Mathematics, Comtemporary Mathematics in Context Course I, II, III	Glencoe McGraw-Hll	2008	Integrated 1, 2, 3	Text Based	Susan Arnold or Jim Coulon	Susan_arnold@mcgraw-hill.com Jim_coulon@mcgraw-hill.com	360/281-2500 Or 760/918-7917
CPM High School Connections Series	CPM Educational Program	Algebra 1: 2006 Algebra 2: 2009	Algebra 1 and 2	Text Based	Brian Hoey	hoey@cpm.org	916/391-3301
CPM High School Connections Series	CPM Educational Program	2007	Geometry	Text Based	Brian Hoey	hoey@cpm.org	916/391-3301
Discovering	Key Curriculum Press	Algebra: 2007	Algebra 1 and 2	Text Based	Kortnii	kjohnson@keypress.com	800/995-

Program Name	Publisher Name	Copyright Date	Course and/or Course Series to be Reviewed	Type of Program	Contact Name	Email	Phone Number
Algebra/Advanced Algebra		Advanced Algebra: 2004			Johnson		6284 ext. 253
Discovering Geometry	Key Curriculum Press	2008	Geometry	Text Based	Kortnii Johnson	kjohnson@keypress.com	800/995-6284 ext. 253
Glencoe McGraw-Hill Algebra 1 and 2	Glencoe McGraw-Hll	2010	Algebra 1 and 2	Text Based	Susan Arnold	Susan_arnold@mcgraw-hill.com	360/281-2500
Glencoe McGraw-Hill Geometry	Glencoe McGraw-Hll	2010	Geometry	Text Based	Susan Arnold	Susan_arnold@mcgraw-hill.com	360/281-2500
Holt Algebra 1 and 2	Holt McDougal	2007	Algebra 1 and 2	Text AND Computer Components	Frank Atkinson	frank.atkinson@hmhpub.com	425/747-7099
Holt Geometry	Holt McDougal	2007	Geometry	Text AND Computer Components	Frank Atkinson	frank.atkinson@hmhpub.com	425/747-7099
Interactive Mathematics Program	Key Curriculum Press	Math I: 2009 Math II: 2004 Math III: 2004	Integrated 1, 2, 3	Text Based	Kortnii Johnson	kjohnson@keypress.com	800/995-6284 ext. 253
MathConnections	It's About Time, Herff Jones Education Division	2006	Algebra 1 and 2	Text Based	Matt Elisara	mpelisara@herffjones.com	360/245-3434
McDougal Littell Algebra 1 and 2	Holt McDougal	2007	Algebra 1 and 2	Text AND Computer Components	Frank Atkinson	frank.atkinson@hmhpub.com	425/747-7099
McDougal Littell Geometry	Holt McDougal	2007	Geometry	Text AND Computer Components	Frank Atkinson	frank.atkinson@hmhpub.com	425/747-7099
Prentice Hall Classics by Foerster	Pearson Prentice Hall	2006	Algebra 1 and 2	Text Based	Dorothy Kulwin or Kyle Bender	dorothy.kulwin@pearson.com kyle.bender@pearson.com	206/819-6814 or 253/906-1059
Prentice Hall Classics by Smith, Charles, etal.	Pearson Prentice Hall	2006	Algebra 1 and 2	Text Based	Dorothy Kulwin or Kyle Bender	dorothy.kulwin@pearson.com kyle.bender@pearson.com	206/819-6814 or 253/906-1059

Program Name	Publisher Name	Copyright Date	Course and/or Course Series to be Reviewed	Type of Program	Contact Name	Email	Phone Number
Prentice Hall Mathematics	Pearson Prentice Hall	2009	Algebra 1 and 2	Text Based	Dorothy Kulwin or Kyle Bender	dorothy.kulwin@pearson.com kyle.bender@pearson.com	206/819-6814 or 253/906-1059
Prentice Hall Mathematics	Pearson Prentice Hall	2009	Geometry	Text Based	Dorothy Kulwin or Kyle Bender	dorothy.kulwin@pearson.com kyle.bender@pearson.com	206/819-6814 or 253/906-1059
SIMMS Integrated Mathematics I, II, III	Kendall/Hunt Publishing	2006	Integrated 1, 2, 3	Text Based	Gloria Hiten	ghiten@kendallhunt.com	877/443-5885

Appendix B. High School Mathematics Standards Organized by Courses

Traditional Sequence	Integrated Sequence						Performance Expectation
Algebra 1	**Math 1**		**Math 2**		**Math 3**		
A1.1.A	M1.1A	A1.1.A					Select and justify functions and equations to model and solve problems.
A1.1.B	M1.1.B	A1.1.B					Solve problems that can be represented by linear functions, equations, and inequalities.
A1.1.C	M1.1.C	A1.1.C					Solve problems that can be represented by a system of two linear equations or inequalities.
A1.1.D			M2.1.C	A1.1.D			Solve problems that can be represented by quadratic functions and equations. (see also A2.1.C)
A1.1.E	M1.1.D	A1.1.E	M2.1.D	A1.1.E			Solve problems that can be represented by exponential functions and equations.
A1.2.A	M1.6.A	A1.2.A					Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.
A1.2.B	M1.6.C	A1.2.B					Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.
A1.2.C	M1.7.C	A1.2.C					Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.
A1.2.D	M1.6.B	A1.2.D					Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.
A1.2.E			M2.5.A	A1.2.E			Use algebraic properties to factor and combine like terms in polynomials.
A1.2.F					M3.6.C	A1.2.F	Add, subtract, multiply, and divide polynomials.
A1.3.A	M1.2.A	A1.3.A					Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.
A1.3.B	M1.2.B	A1.3.B					Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.
A1.3.C	M1.2.C	A1.3.C					Evaluate $f(x)$ at a (i.e., $f(a)$) and solve for x in the equation $f(x) = b$.
A1.4.A	M1.3.A	A1.4.A					Write and solve linear equations and inequalities in one variable.
A1.4.B	M1.3.D	A1.4.B					Write and graph an equation for a line given the slope and the y-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.

Traditional Sequence	Integrated Sequence						Performance Expectation
A1.4.C	M1.3.C	A1.4.C					Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.
A1.4.D	M1.3.E	A1.4.D					Write and solve systems of two linear equations and inequalities in two variables.
A1.4.E	M1.3.B	A1.4.E					Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.
A1.5.A			M2.2.A	A1.5.A			Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.
A1.5.B			M2.2.B	A1.5.B			Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the *x*-intercepts as solutions to a quadratic equation.
A1.5.C			M2.2.D	A1.5.C			Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where *a*, *b*, *c*, and *d* are integers.
A1.5.D			M2.2.F	A1.5.D			Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.
A1.6.A	M1.5.A	A1.6.A					Use and evaluate the accuracy of summary statistics to describe and compare data sets.
A1.6.B	M1.5.C	A1.6.B					Make valid inferences and draw conclusions based on data.
A1.6.C	M1.5.B	A1.6.C					Describe how linear transformations affect the center and spread of univariate data.
A1.6.D	M1.3.F	A1.6.D					Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and *y*-intercept of the line, and use the equation to make predictions.
A1.6.E	M1.3.G	A1.6.E					Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.
A1.7.A	M1.7.A	A1.7.A					Sketch the graph for an exponential function of the form $y = ab^n$ where *n* is an integer, describe the effects that changes in the parameters *a* and *b* have on the graph, and answer questions that arise in situations modeled by exponential functions.
A1.7.B	M1.7.B	A1.7.B					Find and approximate solutions to exponential equations.
A1.7.C	M1.7.D	A1.7.C					Express arithmetic and geometric sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence.
A1.7.D	M1.6.D	A1.7.D					Solve an equation involving several variables by expressing one variable in terms of the others.
A1.8.A	M1.8.A	A1.8.A					Analyze a problem situation and represent it mathematically.

Traditional Sequence	Integrated Sequence						Performance Expectation
A1.8.B	M1.8.B	A1.8.B					Select and apply strategies to solve problems.
A1.8.C	M1.8.C	A1.8.C					Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
A1.8.D	M1.8.D	A1.8.D					Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.
A1.8.E	M1.8.E	A1.8.E					Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
A1.8.F	M1.8.F	A1.8.F					Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
A1.8.G	M1.8.G	A1.8.G					Synthesize information to draw conclusions, and evaluate the arguments and conclusions of others.
A1.8.H					M3.8.H	A1.8.H	Use inductive reasoning about algebra and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.

Traditional Sequence	Integrated Sequence						Performance Expectation
Geometry	**Math 1**		**Math 2**		**Math 3**		
G.1.A	M1.4.A	G.1.A					Distinguish between inductive and deductive reasoning.
G.1.B	M1.4.B	G.1.B					Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.
G.1.C	M1.4.C	G.1.C	M2.3.A	G.1.C			Use deductive reasoning to prove that a valid geometric statement is true.
G.1.D			M2.3.C	G.1.D			Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.
G.1.E			M2.3.B	G.1.E			Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.
G.1.F			M2.3.D	G.1.F			Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.
G.2.A	M1.4.E	G.2.A					Know, prove, and apply theorems about parallel and perpendicular lines.
G.2.B	M1.4.F	G.2.B					Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.
G.2.C	M1.4.G	G.2.C					Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.
G.2.D					M3.5.A	G.2.D	Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.
G.3.A			M2.3.E	G.3.A			Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.
G.3.B	M1.4.D	G.3.B	M2.3.F	G.3.B			Determine and prove triangle congruence, triangle similarity, and other properties of triangles.
G.3.C			M2.3.I	G.3.C			Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems.
G.3.D			M2.3.G	G.3.D			Know, prove, and apply the Pythagorean Theorem and its converse.
G.3.E			M2.3.H	G.3.E			Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.
G.3.F			M2.3.J	G.3.F			Know, prove, and apply basic theorems about parallelograms.
G.3.G			M2.3.K	G.3.G			Know, prove, and apply theorems about properties of quadrilaterals and other polygons.
G.3.H					M3.7.A	G.3.H	Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.
G.3.I					M3.7.C	G.3.I	Explain and perform constructions related to the circle.

Traditional Sequence	Integrated Sequence						Performance Expectation
G.3.J					M3.5.B	G.3.J	Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.
G.3.K					M3.5.C	G.3.K	Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.
G.4.A	M1.3.H	G.4.A					Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.
G.4.B			M2.3.L	G.4.B			Determine the coordinates of a point that is described geometrically.
G.4.C			M2.3.M	G.4.C			Verify and apply properties of triangles and quadrilaterals in the coordinate plane.
G.4.D					M3.7.B	G.4.D	Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).
G.5.A					M3.2.A	G.5.A	Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.
G.5.B					M3.2.B	G.5.B	Determine and apply properties of transformations.
G.5.C					M3.2.C	G.5.C	Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.
G.5.D					M3.2.D	G.5.D	Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.
G.6.A					M3.7.D	G.6.A	Derive and apply formulas for arc length and area of a sector of a circle.
G.6.B					M3.5.F	G.6.B	Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.
G.6.C					M3.5.D	G.6.C	Apply formulas for surface area and volume of three-dimensional figures to solve problems.
G.6.D					M3.5.E	G.6.D	Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.
G.6.E			M2.5.B	G.6.E			Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.
G.6.F			M2.5.C	G.6.F			Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.

Traditional Sequence	Integrated Sequence						Performance Expectation
G.7.A			M2.6.A	G.7.A			Analyze a problem situation and represent it mathematically.
G.7.B			M2.6.B	G.7.B			Select and apply strategies to solve problems.
G.7.C			M2.6.C	G.7.C			Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
G.7.D			M2.6.D	G.7.D			Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.
G.7.E			M2.6.E	G.7.E			Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
G.7.F			M2.6.F	G.7.F			Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
G.7.G			M2.6.G	G.7.G			Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.
G.7.H	M1.8.H	G.7.H	M2.6.H	G.7.H			Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.

Traditional Sequence	Integrated Sequence						Performance Expectation
Algebra 2	**Math 1**		**Math 2**		**Math 3**		
A2.1.A			M2.1.A	A2.1.A	M3.1.A	A2.1.A	Select and justify functions and equations to model and solve problems.
A2.1.B			M2.1.B	A2.1.B	M3.1.B	A2.1.B	Solve problems that can be represented by systems of equations and inequalities.
A2.1.C			M2.1.C	A2.1.C	M3.1.C	A2.1.C	Solve problems that can be represented by quadratic functions, equations, and inequalities.
A2.1.D					M3.1.D	A2.1.D	Solve problems that can be represented by exponential and logarithmic functions and equations.
A2.1.E					M3.1.E	A2.1.E	Solve problems that can be represented by inverse variations of the forms $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx+c)}$.
A2.1.F			M2.1.E	A2.1.F			Solve problems involving combinations and permutations.
A2.2.A					M3.6.A	A2.2.A	Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.
A2.2.B					M3.6.B	A2.2.B	Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.
A2.2.C					M3.6.D	A2.2.C	Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.
A2.3.A			M2.2.C	A2.3.A			Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.
A2.3.B			M2.2.E	A2.3.B			Determine the number and nature of the roots of a quadratic function.
A2.3.C			M2.2.G	A2.3.C			Solve quadratic equations and inequalities, including equations with complex roots.
A2.4.A					M3.3.A	A2.4.A	Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.
A2.4.B					M3.3.B	A2.4.B	Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.
A2.4.C					M3.3.C	A2.4.C	Solve exponential and logarithmic equations.
A2.5.A					M3.2.E	A2.5.A	Construct new functions using the transformations $f(x - h)$, $f(x) + k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).
A2.5.B					M3.3.D	A2.5.B	Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{x-c} + d$, and solve related equations.

Traditional Sequence	Integrated Sequence						Performance Expectation
A2.5.C	M1.2.D	A2.5.C			M3.3.E	A2.5.C	Plot points, sketch, and describe the graphs of functions of the form $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx + c)}$, and solve related equations.
A2.5.D					M3.3.F	A2.5.D	Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.
A2.6.A			M2.4.A	A2.6.A			Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).
A2.6.B			M2.4.B	A2.6.B			Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.
A2.6.C			M2.4.C	A2.6.C			Compute permutations and combinations, and use the results to calculate probabilities.
A2.6.D			M2.4.D	A2.6.D			Apply the binomial theorem to solve problems involving probability.
A2.6.E			M2.2.H	A2.6.E			Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.
A2.6.F					M3.4.A	A2.6.F	Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.
A2.6.G					M3.4.B	A2.6.G	Calculate and interpret margin of error and confidence intervals for population proportions.
A2.7.A					M3.3.G	A2.7.A	Solve systems of three equations with three variables.
A2.7.B			M2.5.D	A2.7.B			Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.
A2.8.A					M3.8.A	A2.8.A	Analyze a problem situation and represent it mathematically.
A2.8.B					M3.8.B	A2.8.B	Select and apply strategies to solve problems.
A2.8.C					M3.8.C	A2.8.C	Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
A2.8.D					M3.8.D	A2.8.D	Generalize a solution strategy for a single problem to a class of related problems and apply a strategy for a class of related problems to solve specific problems.

Traditional Sequence	Integrated Sequence						Performance Expectation
A2.8.E					M3.8.E	A2.8.E	Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
A2.8.F					M3.8.F	A2.8.F	Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
A2.8.G					M3.8.G	A2.8.G	Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.
A2.8.H					M3.8.H	A2.8.H	Use inductive reasoning and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.

Appendix C. Review Instruments

This section shows the content of each of the high school review instruments: Part 1: Content/standards Alignment and Part 2: Other Factors.

Algebra 1		Date:	
Program:		Reviewer #:	

(Rate each item on the scale 0-not met, 1-limited content, 2-limited practice, 3-fully met)							
A1.1. Core Content: Solving problems *(Algebra)*		**0**	**1**	**2**	**3**	**A2**	**Evidence**
A1.1.A	Select and justify functions and equations to model and solve problems.	○	○	○	○	◇	
A1.1.B	Solve problems that can be represented by linear functions, equations, and inequalities.	○	○	○	○	◇	
A1.1.C	Solve problems that can be represented by a system of two linear equations or inequalities.	○	○	○	○	◇	
A1.1.D	Solve problems that can be represented by quadratic functions and equations.	○	○	○	○	◇	
A1.1.E	Solve problems that can be represented by exponential functions and equations.	○	○	○	○	◇	

A1.2. Core Content: Numbers, expressions, and operations *(Numbers, Operations, Algebra)*		**0**	**1**	**2**	**3**	**A2**	**Evidence**
A1.2.A	Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.	○	○	○	○	◇	
A1.2.B	Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.	○	○	○	○	◇	
A1.2.C	Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.	○	○	○	○	◇	
A1.2.D	Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.	○	○	○	○	◇	
A1.2.E	Use algebraic properties to factor and combine like terms in polynomials.	○	○	○	○	◇	
A1.2.F	Add, subtract, multiply, and divide polynomials.	○	○	○	○	◇	

A1.3. Core Content: Characteristics and behaviors of functions *(Algebra)*		**0**	**1**	**2**	**3**	**A2**	**Evidence**
A1.3.A	Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.	○	○	○	○	◇	
A1.3.B	Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.	○	○	○	○	◇	
A1.3.C	Evaluate $f(x)$ at a (i.e., $f(a)$) and solve for x in the equation $f(x) = b$.	○	○	○	○	◇	

A1.4. Core Content: Linear functions, equations, and inequalities *(Algebra)*		0	1	2	3	A2	Evidence
A1.4.A	Write and solve linear equations and inequalities in one variable.	○	○	○	○	◇	
A1.4.B	Write and graph an equation for a line given the slope and the *y*-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.	○	○	○	○	◇	
A1.4.C	Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.	○	○	○	○	◇	
A1.4.D	Write and solve systems of two linear equations and inequalities in two variables.	○	○	○	○	◇	
A1.4.E	Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.	○	○	○	○	◇	

A1.5. Core Content: Quadratic functions and equations *(Algebra)*		0	1	2	3	A2	Evidence
A1.5.A	Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.	○	○	○	○	◇	
A1.5.B	Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the *x*-intercepts as solutions to a quadratic equation.	○	○	○	○	◇	
A1.5.C	Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where *a, b, c*, and *d* are integers.	○	○	○	○	◇	
A1.5.D	Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.	○	○	○	○	◇	

A1.6. Core Content: Data and distributions *(Data/Statistics/Probability)*		0	1	2	3	A2	Evidence
A1.6.A	Use and evaluate the accuracy of summary statistics to describe and compare data sets.	○	○	○	○	◇	
A1.6.B	Make valid inferences and draw conclusions based on data.	○	○	○	○	◇	
A1.6.C	Describe how linear transformations affect the center and spread of univariate data.	○	○	○	○	◇	
A1.6.D	Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and *y*-intercept of the line, and use the equation to make predictions.	○	○	○	○	◇	
A1.6.E	Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.	○	○	○	○	◇	

A1.7. Additional Key Content *(Algebra)*		0	1	2	3	A2	Evidence
A1.7.A	Sketch the graph for an exponential function of the form $y = ab^n$ where *n* is an integer, describe the effects that changes in the parameters *a* and *b* have on the graph, and answer questions that arise in situations modeled by exponential functions.	○	○	○	○	◇	
A1.7.B	Find and approximate solutions to exponential equations.	○	○	○	○	◇	
A1.7.C	Express arithmetic and geometric sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence.	○	○	○	○	◇	
A1.7.D	Solve an equation involving several variables by expressing one variable in terms of the others.	○	○	○	○	◇	

A1.8. Core Processes: Reasoning, problem solving, and communication		0	1	2	3	A2	Evidence
A1.8.A	Analyze a problem situation and represent it mathematically.	○	○	○	○	◇	
A1.8.B	Select and apply strategies to solve problems.	○	○	○	○	◇	
A1.8.C	Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.	○	○	○	○	◇	
A1.8.D	Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.	○	○	○	○	◇	
A1.8.E	Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.	○	○	○	○	◇	
A1.8.F	Summarize mathematical ideas with precision and efficiency for a given audience and purpose.	○	○	○	○	◇	
A1.8.G	Synthesize information to draw conclusions, and evaluate the arguments and conclusions of others.	○	○	○	○	◇	
A1.8.H	Use inductive reasoning about algebra and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.	○	○	○	○	◇	

Geometry		Date:	
Program:		Reviewer #:	

(Rate each item on the scale 0-not met, 1-limited content, 2-limited practice, 3-fully met)						
G.1. Core Content: Logical arguments and proofs *(Logic)*		**0**	**1**	**2**	**3**	**Evidence**
G.1.A	Distinguish between inductive and deductive reasoning.	O	O	O	O	
G.1.B	Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.	O	O	O	O	
G.1.C	Use deductive reasoning to prove that a valid geometric statement is true.	O	O	O	O	
G.1.D	Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.	O	O	O	O	
G.1.E	Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.	O	O	O	O	
G.1.F	Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.	O	O	O	O	

G.2. Core Content: Lines and angles *(Geometry/Measurement)*		**0**	**1**	**2**	**3**	**Evidence**
G.2.A	Know, prove, and apply theorems about parallel and perpendicular lines.	O	O	O	O	
G.2.B	Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.	O	O	O	O	
G.2.C	Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.	O	O	O	O	
G.2.D	Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.	O	O	O	O	

G.3. Core Content: Two- and three-dimensional figures *(Geometry/Measurement)*		0	1	2	3	Evidence
G.3.A	Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.	○	○	○	○	
G.3.B	Determine and prove triangle congruence, triangle similarity, and other properties of triangles.	○	○	○	○	
G.3.C	Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems.	○	○	○	○	
G.3.D	Know, prove, and apply the Pythagorean Theorem and its converse.	○	○	○	○	
G.3.E	Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.	○	○	○	○	
G.3.F	Know, prove, and apply basic theorems about parallelograms.	○	○	○	○	
G.3.G	Know, prove, and apply theorems about properties of quadrilaterals and other polygons.	○	○	○	○	
G.3.H	Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.	○	○	○	○	
G.3.I	Explain and perform constructions related to the circle.	○	○	○	○	
G.3.J	Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.	○	○	○	○	
G.3.K	Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.	○	○	○	○	

G.4. Core Content: Geometry in the coordinate plane *(Geometry/Measurement, Algebra)*		0	1	2	3	Evidence
G.4.A	Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.	○	○	○	○	
G.4.B	Determine the coordinates of a point that is described geometrically.	○	○	○	○	
G.4.C	Verify and apply properties of triangles and quadrilaterals in the coordinate plane.	○	○	○	○	
G.4.D	Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).	○	○	○	○	

G.5. Core Content: Geometric transformations *(Geometry/Measurement)*		0	1	2	3	Evidence
G.5.A	Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.	○	○	○	○	
G.5.B	Determine and apply properties of transformations.	○	○	○	○	
G.5.C	Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.	○	○	○	○	
G.5.D	Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.	○	○	○	○	

G.6. Additional Key Content *(Measurement)*		0	1	2	3	Evidence
G.6.A	Derive and apply formulas for arc length and area of a sector of a circle.	O	O	O	O	
G.6.B	Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.	O	O	O	O	
G.6.C	Apply formulas for surface area and volume of three-dimensional figures to solve problems.	O	O	O	O	
G.6.D	Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.	O	O	O	O	
G.6.E	Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.	O	O	O	O	
G.6.F	Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.	O	O	O	O	

G.7. Core Processes: Reasoning, problem solving, and communication		0	1	2	3	Evidence
G.7.A	Analyze a problem situation and represent it mathematically.	O	O	O	O	
G.7.B	Select and apply strategies to solve problems.	O	O	O	O	
G.7.C	Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.	O	O	O	O	
G.7.D	Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.	O	O	O	O	
G.7.E	Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.	O	O	O	O	
G.7.F	Summarize mathematical ideas with precision and efficiency for a given audience and purpose.	O	O	O	O	
G.7.G	Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.	O	O	O	O	
G.7.H	Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.	O	O	O	O	

Algebra 2

Program:		Date:	
		Reviewer #:	

(Rate each item on the scale 0-not met, 1-limited content, 2-limited practice, 3-fully met)

A2.1. Core Content: Solving problems		0	1	2	3	A1	Evidence
A2.1.A	Select and justify functions and equations to model and solve problems.	○	○	○	○	◇	
A2.1.B	Solve problems that can be represented by systems of equations and inequalities.	○	○	○	○	◇	
A2.1.C	Solve problems that can be represented by quadratic functions, equations, and inequalities.	○	○	○	○	◇	
A2.1.D	Solve problems that can be represented by exponential and logarithmic functions and equations.	○	○	○	○	◇	
A2.1.E	Solve problems that can be represented by inverse variations of the forms $f(x)=a/x+b$, $f(x)=a/x^2+b$, and $f(x)=a/(bx+c)$.	○	○	○	○	◇	
A2.1.F	Solve problems involving combinations and permutations.	○	○	○	○	◇	

A2.2. Core Content: Numbers, expressions, and operations *(Numbers, Operations, Algebra)*		0	1	2	3	A1	Evidence
A2.2.A	Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.	○	○	○	○	◇	
A2.2.B	Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.	○	○	○	○	◇	
A2.2.C	Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.	○	○	○	○	◇	

A2.3. Core Content: Quadratic functions and equations *(Algebra)*		0	1	2	3	A1	Evidence
A2.3.A	Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.	○	○	○	○	◇	
A2.3.B	Determine the number and nature of the roots of a quadratic function.	○	○	○	○	◇	
A2.3.C	Solve quadratic equations and inequalities, including equations with complex roots.	○	○	○	○	◇	

A2.4. Core Content: Exponential and logarithmic functions and equations *(Algebra)*		0	1	2	3	A1	Evidence
A2.4.A	Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.	○	○	○	○	◇	
A2.4.B	Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.	○	○	○	○	◇	
A2.4.C	Solve exponential and logarithmic equations.	○	○	○	○	◇	

A2.5. Core Content: Additional functions and equations *(Algebra)*		0	1	2	3	A1	Evidence
A2.5.A	Construct new functions using the transformations $f(x-h)$, $f(x)+k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).	○	○	○	○	◇	
A2.5.B	Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{(x-c)} + d$, and solve related equations.	○	○	○	○	◇	
A2.5.C	Plot points, sketch, and describe the graphs of functions of the form $f(x) = a/x + b$, $f(x) = a/x^2 + b$, and $f(x) = a/(bx + c)$, and solve related equations.	○	○	○	○	◇	
A2.5.D	Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.	○	○	○	○	◇	

A2.6. Core Content: Probability, data, and distributions *(Data/Statistics/Probability)*		0	1	2	3	A1	Evidence
A2.6.A	Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).	○	○	○	○	◇	
A2.6.B	Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.	○	○	○	○	◇	
A2.6.C	Compute permutations and combinations, and use the results to calculate probabilities.	○	○	○	○	◇	
A2.6.D	Apply the binomial theorem to solve problems involving probability.	○	○	○	○	◇	
A2.6.E	Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.	○	○	○	○	◇	
A2.6.F	Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.	○	○	○	○	◇	
A2.6.G	Calculate and interpret margin of error and confidence intervals for population proportions.	○	○	○	○	◇	

A2.7. Additional Key Content *(Algebra)*		0	1	2	3	A1	Evidence
A2.7.A	Solve systems of three equations with three variables.	○	○	○	○	◇	
A2.7.B	Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.	○	○	○	○	◇	

A2.8. Core Processes: Reasoning, problem solving, and communication		0	1	2	3	A1	Evidence
A2.8.A	Analyze a problem situation and represent it mathematically.	○	○	○	○	◇	
A2.8.B	Select and apply strategies to solve problems.	○	○	○	○	◇	
A2.8.C	Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.	○	○	○	○	◇	
A2.8.D	Generalize a solution strategy for a single problem to a class of related problems and apply a strategy for a class of related problems to solve specific problems.	○	○	○	○	◇	
A2.8.E	Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.	○	○	○	○	◇	
A2.8.F	Summarize mathematical ideas with precision and efficiency for a given audience and purpose.	○	○	○	○	◇	
A2.8.G	Use inductive reasoning and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.	○	○	○	○	◇	
A2.8.H	Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.	○	○	○	○	◇	

Mathematics 1		Date:	
Program:		Reviewer #:	

(Rate each item on the scale 0-not met, 1-limited content, 2-limited practice, 3-fully met)								
M1.1. Core Content: Solving problems *(Algebra)*		**0**	**1**	**2**	**3**	**M2**	**M3**	**Evidence**
M1.1.A	Select and justify functions and equations to model and solve problems.	○	○	○	○	◇	◇	
M1.1.B	Solve problems that can be represented by linear functions, equations, and inequalities.	○	○	○	○	◇	◇	
M1.1.C	Solve problems that can be represented by a system of two linear equations or inequalities.	○	○	○	○	◇	◇	
M1.1.D	Solve problems that can be represented by exponential functions and equations.	○	○	○	○	◇	◇	

M1.2. Core Content: Characteristics and behaviors of functions *(Algebra)*		**0**	**1**	**2**	**3**	**M2**	**M3**	**Evidence**
M1.2.A	Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.	○	○	○	○	◇	◇	
M1.2.B	Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.	○	○	○	○	◇	◇	
M1.2.C	Evaluate $f(x)$ at a (i.e., $f(a)$) and solve for x in the equation $f(x) = b$.	○	○	○	○	◇	◇	
M1.2.D	Plot points, sketch, and describe the graphs of functions of the form $f(x) = a/x + b$.	○	○	○	○	◇	◇	

M1.3 Core Cont.: Linear funcs., equations, and relationships *(Alg., Geom./Meas., Data/Stats./Prob.)*		**0**	**1**	**2**	**3**	**M2**	**M3**	**Evidence**
M1.3.A	Write and solve linear equations and inequalities in one variable.	○	○	○	○	◇	◇	
M1.3.B	Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.	○	○	○	○	◇	◇	
M1.3.C	Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.	○	○	○	○	◇	◇	
M1.3.D	Write and graph an equation for a line given the slope and the y-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.	○	○	○	○	◇	◇	
M1.3.E	Write and solve systems of two linear equations and inequalities in two variables.	○	○	○	○	◇	◇	
M1.3.F	Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and y-intercept of the line, and use the equation to make predictions.	○	○	○	○	◇	◇	
M1.3.G	Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.	○	○	○	○	◇	◇	
M1.3.H	Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.	○	○	○	○	◇	◇	

M1.4. Core Content: Proportionality, similarity, and geometric reasoning *(Geometry/Measurement)*		0	1	2	3	M2	M3	Evidence
M1.4.A	Distinguish between inductive and deductive reasoning.	○	○	○	○	◇	◇	
M1.4.B	Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.	○	○	○	○	◇	◇	
M1.4.C	Use deductive reasoning to prove that a valid geometric statement is true.	○	○	○	○	◇	◇	
M1.4.D	Determine and prove triangle similarity.	○	○	○	○	◇	◇	
M1.4.E	Know, prove, and apply theorems about parallel and perpendicular lines.	○	○	○	○	◇	◇	
M1.4.F	Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.	○	○	○	○	◇	◇	
M1.4.G	Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.	○	○	○	○	◇	◇	

M1.5. Core Content: Data and distributions *(Data/Statistics/Probability)*		0	1	2	3	M2	M3	Evidence
M1.5.A	Use and evaluate the accuracy of summary statistics to describe and compare data sets.	○	○	○	○	◇	◇	
M1.5.B	Describe how linear transformations affect the center and spread of univariate data.	○	○	○	○	◇	◇	
M1.5.C	Make valid inferences and draw conclusions based on data.	○	○	○	○	◇	◇	

M1.6. Core Content: Numbers, expressions, and operations *(Numbers, Operations, Algebra)*		0	1	2	3	M2	M3	Evidence
M1.6.A	Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.	○	○	○	○	◇	◇	
M1.6.B	Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.	○	○	○	○	◇	◇	
M1.6.C	Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.	○	○	○	○	◇	◇	
M1.6.D	Solve an equation involving several variables by expressing one variable in terms of the others.	○	○	○	○	◇	◇	

M1.7. Additional Key Content *(Numbers, Algebra)*		0	1	2	3	M2	M3	Evidence
M1.7.A	Sketch the graph for an exponential function of the form $y = ab^n$ where n is an integer, describe the effects that changes in the parameters a and b have on the graph, and answer questions that arise in situations modeled by exponential functions.	○	○	○	○	◇	◇	
M1.7.B	Find and approximate solutions to exponential equations.	○	○	○	○	◇	◇	
M1.7.C	Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.	○	○	○	○	◇	◇	
M1.7.D	Express arithmetic and geometric sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence.	○	○	○	○	◇	◇	

M1.8. Core Processes: Reasoning, problem solving, and communication		0	1	2	3	M2	M3	Evidence
M1.8.A	Analyze a problem situation and represent it mathematically.	○	○	○	○	◇	◇	
M1.8.B	Select and apply strategies to solve problems.	○	○	○	○	◇	◇	
M1.8.C	Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.	○	○	○	○	◇	◇	
M1.8.D	Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.	○	○	○	○	◇	◇	
M1.8.E	Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.	○	○	○	○	◇	◇	
M1.8.F	Summarize mathematical ideas with precision and efficiency for a given audience and purpose.	○	○	○	○	◇	◇	
M1.8.G	Synthesize information to draw conclusions, and evaluate the arguments and conclusions of others.	○	○	○	○	◇	◇	
M1.8.H	Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.	○	○	○	○	◇	◇	

Mathematics 2		Date:	
Program:		Reviewer #:	

(Rate each item on the scale 0-not met, 1-limited content, 2-limited practice, 3-fully met)								
M2.1. Core Content: Modeling situations and solving problems *(Algebra)*		**0**	**1**	**2**	**3**	**M1**	**M3**	**Evidence**
M2.1.A	Select and justify functions and equations to model and solve problems.	○	○	○	○	◇	◇	
M2.1.B	Solve problems that can be represented by systems of equations and inequalities.	○	○	○	○	◇	◇	
M2.1.C	Solve problems that can be represented by quadratic functions, equations, and inequalities.	○	○	○	○	◇	◇	
M2.1.D	Solve problems that can be represented by exponential functions and equations.	○	○	○	○	◇	◇	
M2.1.E	Solve problems involving combinations and permutations.	○	○	○	○	◇	◇	

M2.2. Core Content: Quadratic functions, equations, and relationships *(Algebra)*		**0**	**1**	**2**	**3**	**M1**	**M3**	**Evidence**
M2.2.A	Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.	○	○	○	○	◇	◇	
M2.2.B	Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the *x*-intercepts as solutions to a quadratic equation.	○	○	○	○	◇	◇	
M2.2.C	Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.	○	○	○	○	◇	◇	
M2.2.D	Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where *a, b, c,* and *d* are integers.	○	○	○	○	◇	◇	
M2.2.E	Determine the number and nature of the roots of a quadratic function.	○	○	○	○	◇	◇	
M2.2.F	Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.	○	○	○	○	◇	◇	
M2.2.G	Solve quadratic equations and inequalities, including equations with complex roots.	○	○	○	○	◇	◇	
M2.2.H	Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.	○	○	○	○	◇	◇	

M2.3. Core Content: Conjectures and proofs *(Algebra, Geometry/Measurement)*		0	1	2	3	M1	M3	Evidence
M2.3.A	Use deductive reasoning to prove that a valid geometric statement is true.	○	○	○	○	◇	◇	
M2.3.B	Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.	○	○	○	○	◇	◇	
M2.3.C	Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.	○	○	○	○	◇	◇	
M2.3.D	Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.	○	○	○	○	◇	◇	
M2.3.E	Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.	○	○	○	○	◇	◇	
M2.3.F	Determine and prove triangle congruence and other properties of triangles.	○	○	○	○	◇	◇	
M2.3.G	Know, prove, and apply the Pythagorean Theorem and its converse.	○	○	○	○	◇	◇	

		0	1	2	3	M1	M3	Evidence
M2.3.H	Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.	○	○	○	○	◇	◇	
M2.3.I	Use the properties of special right triangles (30°–60°–90° and 45°– 45°–90°) to solve problems.	○	○	○	○	◇	◇	
M2.3.J	Know, prove, and apply basic theorems about parallelograms.	○	○	○	○	◇	◇	
M2.3.K	Know, prove, and apply theorems about properties of quadrilaterals and other polygons.	○	○	○	○	◇	◇	
M2.3.L	Determine the coordinates of a point that is described geometrically.	○	○	○	○	◇	◇	
M2.3.M	Verify and apply properties of triangles and quadrilaterals in the coordinate plane.	○	○	○	○	◇	◇	

M2.4. Core Content: Probability *(Data/Statistics/Probability)*		0	1	2	3	M1	M3	Evidence
M2.4.A	Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).	○	○	○	○	◇	◇	
M2.4.B	Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.	○	○	○	○	◇	◇	
M2.4.C	Compute permutations and combinations, and use the results to calculate probabilities.	○	○	○	○	◇	◇	
M2.4.D	Apply the binomial theorem to solve problems involving probability.	○	○	○	○	◇	◇	

M2.5. Additional Key Content *(Algebra, Measurement)*		0	1	2	3	M1	M3	Evidence
M2.5.A	Use algebraic properties to factor and combine like terms in polynomials.	○	○	○	○	◇	◇	
M2.5.B	Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.	○	○	○	○	◇	◇	
M2.5.C	Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.	○	○	○	○	◇	◇	
M2.5.D	Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.	○	○	○	○	◇	◇	

M2.6. Core Processes: Reasoning, problem solving, and communication		0	1	2	3	M1	M3	Evidence
M2.6.A	Analyze a problem situation and represent it mathematically.	○	○	○	○	◇	◇	
M2.6.B	Select and apply strategies to solve problems.	○	○	○	○	◇	◇	
M2.6.C	Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.	○	○	○	○	◇	◇	
M2.6.D	Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.	○	○	○	○	◇	◇	
M2.6.E	Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.	○	○	○	○	◇	◇	
M2.6.F	Summarize mathematical ideas with precision and efficiency for a given audience and purpose.	○	○	○	○	◇	◇	
M2.6.G	Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.	○	○	○	○	◇	◇	
M2.6.H	Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.	○	○	○	○	◇	◇	

Mathematics 3		Date:	
Program:		Reviewer #:	

(Rate each item on the scale 0-not met, 1-limited content, 2-limited practice, 3-fully met)

M3.1. Core Content: Solving problems *(Algebra)*		0	1	2	3	M1	M2	Evidence
M3.1.A	Select and justify functions and equations to model and solve problems.	○	○	○	○	◇	◇	
M3.1.B	Solve problems that can be represented by systems of equations and inequalities.	○	○	○	○	◇	◇	
M3.1.C	Solve problems that can be represented by quadratic functions, equations, and inequalities.	○	○	○	○	◇	◇	
M3.1.D	Solve problems that can be represented by exponential and logarithmic functions and equations.	○	○	○	○	◇	◇	
M3.1.E	Solve problems that can be represented by inverse variations of the forms $f(x) = a/x + b$, $f(x) = a/x^2 + b$, and $f(x) = a/(bx + c)$.	○	○	○	○	◇	◇	

M3.2. Core Content: Transformations and functions *(Algebra, Geometry/Measurement)*		0	1	2	3	M1	M2	Evidence
M3.2.A	Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.	○	○	○	○	◇	◇	
M3.2.B	Determine and apply properties of transformations.	○	○	○	○	◇	◇	
M3.2.C	Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.	○	○	○	○	◇	◇	
M3.2.D	Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.	○	○	○	○	◇	◇	
M3.2.E	Construct new functions using the transformations $f(x – h)$, $f(x) + k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).	○	○	○	○	◇	◇	

M3.3. Core Content: Functions and modeling *(Algebra)*		0	1	2	3	M1	M2	Evidence
M3.3.A	Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.	○	○	○	○	◇	◇	
M3.3.B	Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.	○	○	○	○	◇	◇	
M3.3.C	Solve exponential and logarithmic equations.	○	○	○	○	◇	◇	
M3.3.D	Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{(x – c)} + d$, and solve related equations.	○	○	○	○	◇	◇	
M3.3.E	Plot points, sketch, and describe the graphs of functions of the form $f(x) = a/x^2 + b$ and $f(x) = a/(bx + c)$, and solve related equations.	○	○	○	○	◇	◇	
M3.3.F	Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.	○	○	○	○	◇	◇	
M3.3.G	Solve systems of three equations with three variables.	○	○	○	○	◇	◇	

M3.4. Core Content: Quantifying variability *(Data/Statistics/Probability)*		0	1	2	3	M1	M2	Evidence
M3.4.A	Calculate and interpret measures of variability and std. deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.	○	○	○	○	◇	◇	
M3.4.B	Calculate and interpret margin of error and confidence intervals for population proportions.	○	○	○	○	◇	◇	

M3.5. Core Content: Three-dimensional geometry *(Geometry/Measurement)*		0	1	2	3	M1	M2	Evidence
M3.5.A	Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.	○	○	○	○	◇	◇	
M3.5.B	Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.	○	○	○	○	◇	◇	
M3.5.C	Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.	○	○	○	○	◇	◇	
M3.5.D	Apply formulas for surface area and volume of three-dimensional figures to solve problems.	○	○	○	○	◇	◇	
M3.5.E	Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.	○	○	○	○	◇	◇	
M3.5.F	Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.	○	○	○	○	◇	◇	

M3.6. Core Content: Algebraic properties *(Numbers, Algebra)*		0	1	2	3	M1	M2	Evidence
M3.6.A	Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.	○	○	○	○	◇	◇	
M3.6.B	Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.	○	○	○	○	◇	◇	
M3.6.C	Add, subtract, multiply, and divide polynomials.	○	○	○	○	◇	◇	
M3.6.D	Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.	○	○	○	○	◇	◇	

M3.7. Additional Key Content *(Geometry/Measurement)*		0	1	2	3	M1	M2	Evidence
M3.7.A	Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.	○	○	○	○	◇	◇	
M3.7.B	Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).	○	○	○	○	◇	◇	
M3.7.C	Explain and perform constructions related to the circle.	○	○	○	○	◇	◇	
M3.7.D	Derive and apply formulas for arc length and area of a sector of a circle.	○	○	○	○	◇	◇	

M3.8. Core Processes: Reasoning, problem solving, and communication		0	1	2	3	M1	M2	Evidence
M3.8.A	Analyze a problem situation and represent it mathematically.	○	○	○	○	◇	◇	
M3.8.B	Select and apply strategies to solve problems.	○	○	○	○	◇	◇	
M3.8.C	Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.	○	○	○	○	◇	◇	
M3.8.D	Generalize a solution strategy for a single problem to a class of related problems and apply a strategy for a class of related problems to solve specific problems.	○	○	○	○	◇	◇	
M3.8.E	Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.	○	○	○	○	◇	◇	
M3.8.F	Summarize mathematical ideas with precision and efficiency for a given audience and purpose.	○	○	○	○	◇	◇	
M3.8.G	Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.	○	○	○	○	◇	◇	
M3.8.H	Use inductive reasoning and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.	○	○	○	○	◇	◇	

Math Instructional Materials Review – Other Factors

(Rate each item on the scale of 1-Strongly disagree, 2.-Disagree, 3-Agree, 4-Strongly agree)

Grade:		Date:	
Program:		Reviewer #:	

	Program Organization and Design	**Strongly disagree**	**disagree**	**agree**	**Strongly agree**
1.	The content has a coherent and well-developed sequence (organized to promote student learning, links facts and concepts in a way that supports retrieval, builds from & extends concepts previously developed, strongly connects concepts to overarching framework)	O	O	O	O
2.	Program includes a balance of skill-building, conceptual understanding, and application	O	O	O	O
3.	Tasks are varied: some have one correct and verifiable answer; some are of an open nature with multiple solutions	O	O	O	O
4.	The materials help promote classroom discourse	O	O	O	O
5.	The program is organized into units, modules or other structure so that students have sufficient time to develop in-depth major mathematical ideas	O	O	O	O
6.	The instructional materials provide for the use of technology which reflects 21st century ideals for a future-ready student	O	O	O	O
7.	Instructional materials include mathematically accurate and complete indexes and tables of contents to locate specific topics or lessons	O	O	O	O
8.	The materials have pictures that match the text in close proximity, with few unrelated images	O	O	O	O
9.	Materials are concise and balance contextual learning with brevity	O	O	O	O
10.	Content is developed for conceptual understanding: (limited number of key concepts, in-depth development at appropriate age level)	O	O	O	O

	Student Learning	**1**	**2**	**3**	**4**
1.	Tasks lead to conceptual development of core content, procedural fluency, and core processes abilities including solving non-routine problems	O	O	O	O
2.	Tasks build upon prior knowledge	O	O	O	O
3.	Tasks lead to problem solving for abstract, real-world and non-routine problems	O	O	O	O
4.	Tasks encourage students to think about their own thinking	O	O	O	O
5.	The program provides opportunities to develop students' computational fluency using brain power without use of calculators	O	O	O	O
6.	Tasks occasionally use technology to deal with messier numbers or help the students see the math with graphical displays	O	O	O	O
7.	The program promotes understanding and fluency in number sense and operations	O	O	O	O
8.	The program leads students to mastery of rigorous multiple-step word problems	O	O	O	O
9.	The materials develop students' use of standard mathematics terminology/vocabulary	O	O	O	O
10.	Objectives are written for students	O	O	O	O

	Instructional Planning and Professional Support	Strongly disagree	disagree	agree	Strongly agree
1.	The instructional materials provide suggestions to teachers on how to help students access prior learning as a foundation for further math learning	O	O	O	O
2.	The instructional materials provide suggestions to teachers on how to help students learn to conjecture, reason, generalize and solve problems	O	O	O	O
3.	The instructional materials provide suggestions to teachers on how to help students connect mathematics ideas and applications to other math topics, other disciplines and real world context	O	O	O	O
4.	Background mathematics information is included so that the concept is explicit in the teacher guide	O	O	O	O
5.	Instructional materials help teachers anticipate and surface common student misconceptions in the moment	O	O	O	O
6.	The materials support a balanced methodology	O	O	O	O
7.	Math concepts are addressed in a context-rich setting (giving examples in context, for instance)	O	O	O	O
8.	Teacher's guides are clear and concise with easy to understand instructions	O	O	O	O

	Assessment	1	2	3	4
1.	The program provides regular assessments to guide student learning	O	O	O	O
2.	There are opportunities for student self-assessment of learning	O	O	O	O
3.	Assessments reflect content, procedural, and process goals and objectives	O	O	O	O
4.	The program includes assessments with multiple purposes (formative, summative and diagnostic)	O	O	O	O
5.	Assessments include multiple choice, short answer and extended response formats.	O	O	O	O
6.	Recommended rubrics or scoring guidelines accurately reflect learning objectives	O	O	O	O
7.	Recommended rubrics or scoring guidelines identify possible student responses both correct & incorrect	O	O	O	O
8.	Accurate answer keys are provided	O	O	O	O

	Equity and Access	1	2	3	4
1.	The program provides methods and materials for differentiating instruction (students with disabilities, gifted/talented, ELL, disadvantaged)	O	O	O	O
2.	Materials support intervention strategies	O	O	O	O
3.	Materials, including assessments are unbiased and relevant to diverse cultures	O	O	O	O
4.	Materials are available in a variety of languages	O	O	O	O
5.	The program includes easily accessible materials which help families to become active participants in their students' math education (e.g. "How You Can Help at Home" letters with explanations, key ideas & vocabulary for each unit, free or inexpensive activities which can be done at home, ideas for community involvement)	O	O	O	O
6.	The program includes guidance and examples to allow students with little home support to be self-sufficient and successful	O	O	O	O

Appendix D. Acknowledgements

Hundreds of people contributed toward the success of the project. Many are listed below. We wish to acknowledge countless others who provided input into the process – parents, teachers, district administrators, business and technical leaders, mathematicians, and other concerned individuals who shared their ideas and feedback on the process and results.

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Dr. George Bright and Dr. Jim King led the mathematical soundness analysis of the top ranked programs in Algebra, Geometry and Integrated Math.

Relevant Strategies staff Nicole Carnegie provided the bulk of the statistical analysis. Eugene Ryser coordinated the data collection process. Dr. June Morita provided expert analysis on the statistical methods. Porsche Everson was the lead author and contributed to the statistical analysis.

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